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Preservice teachers' use of contrasting cases in mathematics instruction

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Abstract Drawing comparisons between students' alternative solution strategies to a single mathematics problem is a powerful yet challenging instructional practice. We examined 80 preservice teachers' when asked to design a short lesson when given a problem and two student solutions—one correct and one incorrect. These micro-teaching events were videotaped and coded, revealing that fewer than half of participants (43%) made any explicit comparison or contrasts between the two solution strategies. Those who did were still not likely to use additional support strategies to draw students' attention to key elements of the comparison. Further, correlations suggest that participants' mathematical content knowledge may be related to whether participants' showed contrasting cases but not to whether they used specific pedagogical cues to support those comparisons. While these micro-teaching events differ from the interactive constraints of a classroom, they reveal that participants did not immediately orient toward differing student solutions as a discussion opportunity, and that future instruction on contrasting cases must highlight the utility of this practice.

Keywords Teacher cognition and practices · Professional development · Mathematics education

Introduction

Contrasting multiple student solution strategies to a single mathematics problem is a powerful instructional practice (Booth et al. 2013; Carpenter et al. 1999; Durkin and Rittle-Johnson 2012; Rittle-Johnson and Star 2007; Smith and Stein 2011; Siegler 2002; Star

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et al. 2015). Comparing and contrasting is posited to foster expert-like, flexible, and conceptual mathematical knowledge (e.g., Alfieri et al. 2013; English 1997, 2004; Kilpatrick et al. 2001; National Mathematics Advisory Panel 2008), partly by supporting explanation, developing more integrative mathematical content knowledge, or reducing use of misconceptions. The recommendation to compare, contrast, and draw connections between mathematical arguments or representations is evident in the Common Core Mathematics Standards within several of the practice standards, as well as in the more integrative content standards (Common Core State Standards Initiative 2010).

Though much empirical support has been provided on the use of contrasting cases in students' mathematics learning (e.g., Alfieri et al 2013; Rittle-Johnson and Star 2007) research suggests that mathematics teachers tend to use comparisons and contrasting cases regularly, but often provide too few pedagogical supports to ensure that students are attentive to the meaning of the contrasting cases and draw the connections intended by the instructor (Hiebert et al. 2003; Richland et al. 2007). There is much evidence from the laboratory from the last decades to show that people often misinterpret or fail to notice analogies (e.g., Gick and Holyoak 1980, 1983; Ross 1987), and that adding pedagogical cues that draw attention to key correspondences and relationships improves learning (Alfieri et al. 2013; Begolli and Richland 2016; Richland and McDonough 2010; Rittle-Johnson and Star 2007).

However, there has been very little attention to how teachers orient toward opportunities to contrast cases as well as use strategies to help students make those intended contrasts. We aimed to first understand whether these preservice teacher candidates as a group tended to make comparisons where multiple, different student solutions were available, or preferentially taught a problem without engaging in comparing the solutions at all. Second, we sought to understand how often these candidates who did make comparisons also used pedagogical cues to support those comparisons, such as linking gestures, visual representations, and spatial alignment. Finally, we sought to understand whether participants' mathematical content knowledge related to their propensity to use contrasting cases and support those comparisons.

These data are quite important for the design of teacher preparation programs that aims to disseminate the rich literature on comparison and contrasting cases. If these teacher candidates have very little awareness of comparisons, and tend not to use such comparisons or supports when available, this means that a teacher education program must begin at a more clear and introductory starting point than if these candidates tend to immediately gravitate to using comparison and contrast when given the time to plan a lesson around a problem in which they have multiple student solutions available.

Literature review

Comparing solution strategies

The current study focuses on engaging students in contrasting correct and incorrect solutions to a single problem (e.g., Carpenter et al. 1999; Stein et al. 2008). Benefits have been shown from the use of comparisons between standard and non-standard solution strategies (e.g. Carpenter et al. 1999; Rittle-Johnson et al. 2009; Rittle-Johnson and Star 2007), between multiple visual representations (Schwartz et al. 2011), and between misconceptions and effective strategies (e.g., Richland and McDonough 2010).

In particular, comparing multiple solutions has been shown to be effective for students with adequate prior knowledge (e.g., Große and Renkl 2006; Rittle-Johnson and Star 2007; Rittle-Johnson et al. 2009), and has been hypothesized to allow for improved self-explanation, broader schema formation, and reduced misconception use. For example, Booth et al. (2013) investigated the effectiveness of presenting correct and incorrect solution strategies to high school Algebra 1 students via a cognitive tutor. In experiment two, students were randomly assigned to one of three conditions: (1) only correct solution strategies, (2) only incorrect solutions, or (3) both correct and incorrect solution strategies. Results indicated that students who were presented with either incorrect solution strategies or both correct and incorrect solution strategies benefited most on measures of conceptual knowledge as compared with students who only saw correct solution strategies. Comparing correct and incorrect solutions can help students recognize that they have chosen the incorrect procedure as well as direct their attention to features of the incorrect procedure that make the procedure incorrect.

At the same time, teachers sometimes express concern that comparing solutions may be challenging or confuse struggling students (Lynch and Star 2014; Silver et al. 2005), and there is reason to be concerned about the potential for confusion among those with low content knowledge (Große and Renkl 2007; Rittle-Johnson et al. 2009). Specifically students with low prior knowledge might not be able to recognize features of the solution that make that solution incorrect. In one experiment, Große and Renkl (2007) examined whether providing students with only correct solutions or a mixture of correct and incorrect solutions fostered near and far transfer. Findings suggested that prior knowledge mattered in how effective providing both correct and incorrect solutions was for transfer and that students with low prior knowledge did not benefit from being presented with correct and incorrect solutions.

Given that students' prior knowledge matters in whether students can take advantage of the benefits of being presented with incorrect solution strategies, and because people are known to often fail to notice analogies when they are not explicitly directed to attend to them (e.g., Gick and Holyoak 1980, 1983), teachers also play a key role in scaffolding students' comparison of correct and incorrect solution strategies. Many studies show that learners often fail to notice or attend to relevant comparisons unless they are given very careful support and scaffolding (e.g., Alfieri et al. 2013; Gick and Holyoak 1980, 1983; Richland and McDonough 2010). In one experiment testing fifth grade students' learning from one of three video-edited versions of the same classroom lesson, Begolli and Richland (2016) found that a mathematics discussion comparing a misconception to an accurate solution led to gains in procedural knowledge, flexibility, and conceptual knowledge, but only when the solutions were made visible simultaneously, which provided a support to help students notice and reference the two solutions that were being compared. Solutions compared only orally led to lower gains in conceptual knowledge, and comparing solutions but making them visible sequentially led to the highest rates of misconceptions and the lowest rates of procedural and conceptual knowledge gains.

A study examining eighth grade mathematics teachers' everyday use of comparisons and visual cues to support them suggested that U.S. teachers provide fewer of such cues than teachers in higher achieving regions (Hiebert et al. 2003; Richland et al. 2007). An examination of a subset of the video data collected as part of the Third International Mathematics and Science Study (TIMSS, Hiebert et al. 2003) revealed that U.S. teachers were found to use as many comparisons as teachers in Hong Kong and Japan, but were less likely to use visual cues to support those comparisons. Coded support cues emerged from teachers' practices and built on theory. They included (1) making the compared representations visible, (2) making them visible simultaneously, (3) making them spatially aligned, and (4) using hand movements (gestures) to go back and forth between the compared objects. In all of these cases, U.S. teachers were less likely to use these supports for each comparison than the Japanese or Hong Kong teachers.

Leading discussions to compare solution strategies is particularly challenging for new teachers (Carpenter et al. 1999; Stein et al. 2008). Stein et al. (2008) noted the high pedagogical demands were associated with teachers building discussions around student explanations, and that teacher experience seemed to support these skills. Teachers must have a clear mathematical goal for these comparisons, and must ensure that while their goal is accomplished students are highly engaged in the discussion. Teachers with more experience leading such discussions, for example those who have received training using the Cognitively Guided Instruction framework, showed gains following years of practice (e.g., Carpenter et al. 1999). At the same time, little is known about preservice teachers, and where teachers begin their careers with regard to their intuitions about how to lead classroom students in comparing student solution strategies.

Mathematical knowledge for teaching

There are several reasons that may explain teacher's practices towards using and supporting contrasting cases. Generally, scholars suggest elements of teacher preparation (see Grossman et al. 2000), teacher beliefs (see Fang 1996; Nespor 1987; Philipp 2007), or the ability to notice student ideas (e.g., Van Es and Sherin 2002) could explain their propensity to use specific pedagogical practices in the classroom. In the present study, we focus on mathematics knowledge for teaching—a teacher's knowledge as it relates to carrying out the tasks of teaching (Ball et al. 2008; Hill et al. 2005, p. 372)—as one explanation for whether preservice teacher's use classroom practices around contrasting cases. Mathematical knowledge for teaching is posited to influence their instructional practices and, in turn, student achievement (Hill et al. 2005). Possibly all preservice teachers would engage and highly support their students in comparisons if they could, but some may not have adequate mathematical knowledge for teaching to go beyond standard solution strategies or to make them feel comfortable engaging with students' incorrect solution strategies. It is possible that teaching practices of comparison are related to teachers' mathematics knowledge in that more mathematical knowledge for teaching would lead preservice teachers to more explicitly represent the deep structure of mathematical representations within their instruction. Additionally, measures of mathematical knowledge for teaching focus explicitly on identifying and responding to student errors. We sought to test whether preservice teachers with higher mathematical knowledge for teaching scores produced more comparisons, or supported those comparisons more, than preservice teachers with lower scores.

The present study

The current study seeks to better understand whether preservice teachers are oriented toward using student solutions as opportunities for discussion and comparison, even when they had not been instructed to do so explicitly. The mathematical domain of algebra was chosen because of its importance as a gatekeeper for students to access higher-level mathematics courses as well as because it serves as a central area of instruction addressed by both primary and secondary teachers. Additionally, algebra represents a content area rife with student errors (Baroody and Ginsburg 1983; Kieran 1981; Knuth et al. 2006). We specifically use micro-teaching events (Hadfield et al. 1998) to understand whether preservice teachers, without explicit instruction, make use of practices around contrasting cases in their practice of the profession. Preservice teachers were given a problem and two different student solutions, one an incorrect solution, and were given time to plan. They were then asked to give short lesson in a micro-teaching event with researchers as their audience. These practice sessions were then examined for their inclusion of those (or additional) solution strategies, and if there was any discussion of the strategies, how the preservice teachers supported their learners in contrasting these solutions. Whereas examining a micro-teaching event is not as authentic as placing preservice teachers into a classroom with students as a method to examine their spontaneous use of student solutions, this methodology provided us with more information than surveys or interviews because preservice teachers' planned behaviors were in fact observed.

Overall our research aims were to provide a description of our sample of preservice teachers' tendencies to compare or not compare student solutions that we provided them in an micro-teaching event prompted by a problem and two different student solutions. Further, we aimed to understand how lesson enactments included pedagogical cues to support these comparisons, when they made them, and to explore whether individual differences in comparison practices were related to mathematical knowledge for teaching. We hypothesized that there would be variability between preservice teachers' use of contrasting cases (H1). Additionally, we sought to understand whether teaching practices of comparison were related to teachers' mathematics knowledge such that more mathematical knowledge for teaching (Hill et al. 2005) was hypothesized to be associated with preservice teachers more explicitly representing the deep structure of mathematical representations within their instruction (H2). Thus we also administered participants the CKT-M assessment of mathematical knowledge for teaching (Hill et al. 2008), which we anticipated would be particularly correlated because it has identifying and responding to student errors as a focus of several questions.

Method

Participants

Eighty participants (preservice primary and preservice mathematics teachers) earning an MAT within a teaching credential program in a major university were recruited from the course Advanced Concepts in Cognition. Historically, this course enrolls approximately half primary mathematics teachers and half secondary mathematics teachers. All participants were in preparation tracks for teaching careers in which they would be teaching mathematics. Approximately half of these participants were just entering the program while the other half had completed one year of a credentialing program (which included a mathematical methods course). All participants were enrolled in a cognition course and had completed sections on cognitive development, knowledge representation, transfer, and the cultural foundations of teaching, but they had not directly learned about contrasting cases or comparison. Participants were given extra credit for their participation in this study. Of the 80 recruited, 74 had valid data on the mathematics knowledge for teaching measure. Six participants did not have valid data due to absence when it was administered.

Materials

Teacher mathematics knowledge test

Participants first took an assessment of their mathematical knowledge for teaching: Form B of the Patterns Functions and Algebra 2006 Content Knowledge for teaching Mathematics (CKT-M) developed by (Hill et al. 2005; Hill et al. 2008). We measured preservice teacher's knowledge using the framework of *Mathematical Knowledge for Teaching* (MKT; see Hill et al. 2005) because teaching mathematics through comparison involves sufficient knowledge of mathematics, the ability to interpret nonstandard or unexpected solutions, and an understanding of how to support student's mathematics skills and problem solving. These are all foci of the CKT-M (Hill et al. 2005). As such, the CKT-M asks participants to answer items identifying appropriate student strategies as well as identifying student errors. Details on the reliability and psychometric properties of the CKT-M are given in Hill et al. (2005).

Micro-teaching prompt

On a different day, participants were individually given a prompt, which was taken from one of the questions on Form B of the Patterns Functions and Algebra 2006 CKT-M measure (see Fig. 1).[1] The question included a description of the equation $y = 2x + 3$ and the prompt to write a scenario which represented this equation. Participants were given two student solutions, one correct and one a misconception. The correct solution was a quantitative scenario that could be captured by the equation $y = 2x + 3$, and the incorrect solution was a scenario in which the student had explained this equation as an exponential function. Participants were then asked to develop a mini-lesson with the aim of helping their students not only understand the problem, but also be able to transfer that knowledge to a new context. They were provided with two students solutions, one correct (the second solution) and one incorrect (the first solution) and whether the participant used these example student solutions was up to him or her. Participants were also told to assume prior knowledge on the part of their students when designing the lesson. Participants were allowed as long as they needed to plan for the lesson and instructed that the lesson itself should take no more than 15 min. A blank page was provided to participants to make lesson-planning notes. Participants were allowed to interact with the researchers in the audience who posed as students; however researchers were instructed not to respond to keep the interactions between researchers and participants as consistent across participants as possible.

Procedure

Participation in this study took place over two days. On the first day, participants were asked to complete the Patterns Functions and Algebra 2006 CKT-M measure. This assessment was administered in a timed within-class setting where participants worked independently for 25 min to complete it—the standard time for this measure. On the second day, participants came independently to a small room that was equipped with a white board, white board markers, two chairs, and a video camera, with researchers present as audience. These lessons were recorded with a video camera and then later coded.

[1] Please note that because the prompt was originally an item on the CKT-M assessment form b, the student solutions cannot be provided publicly. For more information see Hill et al. (2005, 2008).

Preservice teacher participants were given a worksheet with the following prompt:

Ms. Ashton was teaching her students to represent situations with algebra. She asked her students to create a story or context that would be appropriately modeled by the equation $y = 2x + 3$.

The participants were then given two alternative student solutions, one correct and one incorrect. (Note: because this prompt was originally an item on the CKT-M assessment form b, the student solutions cannot be provided publicly).

In this lesson, your aim is that students understand the problem, as well as to be able to solve new problems that relied on the same concept but might look different (transfer problems). The way that you use your students' responses is your choice. You can decide to respond to your students' different answers explicitly or not at all.

Fig. 1 Prompt that was used from the CKT-M Measure, given to participants as a prompt for their approximation of practice opportunity

Coding

Videos were coded by two research assistants (coders overlapped on 20% of the videos) to analyze the teacher candidates' enactments of their lessons, specifically collecting data on their use of teaching practices known to support students in drawing mathematical connections through comparing and contrasting cases (based on Richland et al. 2007; Richland and McDonough 2010). We used binary qualitative codes to obtain quantitative data about the frequencies of preservice teachers' use of these practices. Videos were coded with either a one or a zero if the practice was present or not. Cohen's Kappa between raters who coded the videos was 0.80. When coder discrepancies were identified, they were resolved through discussion. Codes fell into three main categories: (1) using visual representations of the two solution strategies (e.g., did participants write the step-by-step reasoning process on the board to reach both solution strategies), (2) using linking gestures—hand movements that moved between the representations (Alibali et al. 2014; Richland et al. 2006), and (3) engaging their imaginary audience in a verbal, explicit comparison (e.g., asking students which solution is preferable and giving a judgment about which solution is preferable). A full list of the codes and their correlations are presented in Table 1.

Results

Frequency counts were conducted for each coded practice and results are presented in three separate sections for ease of interpretation. In the first section, we present overall results of all participants who conducted the video portion of the study (N = 80). In the second section, we reduce the sample to only participants who presented (either verbally, written, or both) both solution strategies that were give in the prompt (n = 34). In the third section, we present results for participants who may not have developed a comparison between the solution strategies, but did make a comparison between the original problem and a new problem of their creation (n = 22). While comparing solutions was the focus of our study, we were interested in whether participants were attuned to comparisons between problems and between solutions. Finally we present results from the examination of the mathematical knowledge for teaching measure (n = 74).

Use of comparison between solution strategies

In the initial analysis of 80 participants (Table 2), fewer than half of the preservice teachers presented (either verbally, visually, or both) the two student solutions they were provided with in the instructed prompt (43%, 34 participants). The incorrect solution was verbalized

Table 1 Correlations for Preservice Teacher Practices of Supporting Comparing Contrasting Cases

	1	2	3	4	5	6	7	8
(1) Total number correct on CKT-M test	1.00							
(2) High CKT-M	0.80***	1.00						
(3) Participant verbalized both of the solutions?	0.16	0.27*	1.00					
(4) Participant verbalizes the step-by-step reasoning for the incorrect strategy?	0.20	0.28*	0.80***	1.00				
(5) Participant verbalizes the step-by-step reasoning for the correct solution strategy?	−0.01	0.15	0.60***	0.27*	1.00			
(6) Participant writes the step-by-step reasoning/calculation incorrect solution	0.16	0.19	0.62***	0.82***	0.18	1.00		
(7) Participant writes the step-by-step reasoning/calculation correct solution	0.10	0.15	0.44***	0.25*	0.69***	0.26*	1.00	
(8) Participant explains reasoning/calculation processes to solve a new problem	0.06	0.00	−0.25*	−0.33**	−0.11	−0.26*	−0.10	1.00
(9) Whole step-by-step reasoning for more than one solution on the board together	0.16	0.19	0.43***	0.35**	0.25*	0.25*	0.29**	0.06
(10) Participant moves hand directly from one solution to another	0.02	0.02	0.32**	0.27*	0.20	0.14	0.11	0.11
(11) Participant asks class which solution they think is preferable	0.06	0.05	0.33**	0.25*	0.25*	0.16	0.17	−0.22
(12) Participant gives judgment about which solution is preferable	0.12	0.18	0.35**	0.42***	0.09	0.39***	0.04	-0.02
(13) Participant asks class how the solutions are similar or different	0.14	0.09	0.36***	0.27*	0.18	0.12	0.05	-0.06
(14) Participant gives statement about why or how the presented solutions are similar or different	0.07	0.06	0.46***	0.31**	0.38***	0.33**	0.31**	0.04
(15) Participant asks class how the presented solutions are different than a new problem	0.00	−0.02	−0.17	−0.08	−0.28*	−0.04	−0.21	0.03
(16) Participant presents an additional solution to problem and explains how solutions are similar or different	0.07	0.09	0.06	−0.03	0.18	−0.03	0.20	0.53***

Table 1 continued

	9	10	11	12	13	14	15
(1) Total number correct on CKT-M test							
(2) High CKT-M							
(3) Participant verbalized both of the solutions?							
(4) Participant verbalizes the step-by-step reasoning for the incorrect strategy?							
(5) Participant verbalizes the step-by-step reasoning for the correct solution strategy?							
(6) Participant writes the step-by-step reasoning/calculation incorrect solution							
(7) Participant writes the step-by-step reasoning/calculation correct solution							
(8) Participant explains reasoning/calculation processes to solve a new problem							
(9) Whole step-by-step reasoning for more than one solution on the board together	1.00						
(10) Participant moves hand directly from one solution to another	0.71***	1.00					
(11) Participant asks class which solution they think is preferable	0.05	0.01	1.00				
(12) Participant gives judgment about which solution is preferable	0.17	0.15	−0.14	1.00			
(13) Participant asks class how the solutions are similar or different	0.36**	0.30**	0.10	0.01	1.00		
(14) Participant gives statement about why or how the presented solutions are similar or different	0.41***	0.38***	0.04	0.15	0.34**	1.00	
(15) Participant asks class how the presented solutions are different than a new problem	−0.12	0.14	−0.07	0.02	0.12	0.02	1.00
(16) Participant presents an additional solution to problem and explains how solutions are similar or different	0.19	0.14	−0.13	0.01	−0.03	0.18	0.12

N = 74. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2 Frequencies of preservice teacher practices of supporting comparing contrasting cases (N = 80)

Code	Frequency	Standard error	CI lower	CI upper
Participant goes through both of the solutions?	0.43	0.06	0.31	0.54
Participant verbalizes the step-by-step reasoning for the incorrect strategy?	0.54	0.06	0.43	0.65
Participant verbalizes the step-by-step reasoning for the correct solution strategy?	0.68	0.05	0.57	0.78
Participant writes the step-by-step reasoning/calculation for the incorrect strategy	0.44	0.06	0.33	0.55
Participant writes the step-by-step reasoning/calculation for the correct solution	0.54	0.06	0.43	0.65
Participant explains reasoning/calculation processes to solve a new problem	0.28	0.05	0.18	0.37
Whole step-by-step reasoning for more than one solution on the board together	0.28	0.05	0.18	0.37
Participant moves hand directly from one solution to another	0.33	0.05	0.22	0.43
Participant asks class which solution they think is preferable	0.11	0.04	0.04	0.18
Participant gives judgment about which solution is preferable	0.29	0.05	0.19	0.39
Participant asks class how the solutions are similar or different	0.13	0.04	0.05	0.20
Participant gives statement about why or how the presented solutions are similar or different	0.29	0.05	0.19	0.39
Participant asks class how the presented solutions are different than a new problem	0.04	0.02	−0.01	0.08
Participant presents an additional solution to problem and explains how solutions are similar or different	0.13	0.04	0.05	0.20

by just over half of participants (54%) and written on the board by 44% (35 participants). The correct solution was verbalized by 68% (54 participants) of participants, and written by 54% (43 participants) of participants. Teachers who did not draw attention to either student-generated solution strategies in their lesson typically gave a description of the problem itself and modeled a way to produce a solution. Twenty-eight percent (22 participants) of participants explained the reasoning to solve a new problem. As illustrated in the rest of Table 2, practices around comparing the two student solutions was low. Thirty-three percent (26 participants) of these participants gestured between solution strategies, and practices around giving judgments about which solution is preferable and why or how the presented solutions are similar or different occurred at 29% (23 participants). When looking at the percentage of participants who asked the imaginary audience to engage in comparison, the rates are much lower—only 11% (9 participants) of participants asked the imaginary audience which solution they thought was preferable and 13% (10 participants) asked the imaginary audience how the solutions are similar or different from one another. The distribution of the total number of coded practices (out of 14) is shown in Fig. 2.

Enactment of support strategies for participants who compared solutions

To better understand the details of how participants supported the comparisons they included in their practice, we examined the support strategies enacted in a reduced-sample of thirty-four participants who presented (either verbally, visually, or both) both solution

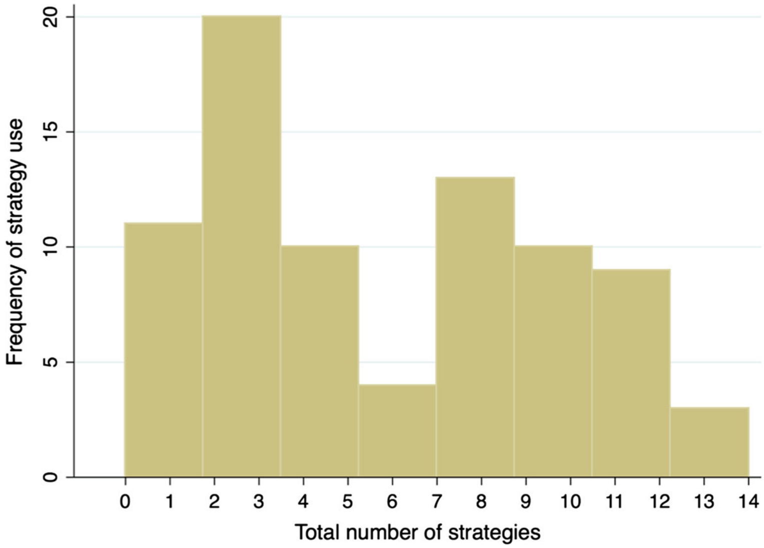

Fig. 2 Total number of coded strategies exhibited by participants

strategies given to participants in the prompt. The full list of frequencies are provided in Table 3. Of those who verbalized both solution strategies, 79% (27 participants) wrote the step-by-step reasoning for both solutions on the board. A smaller percentage (50%, 17 participants) made both solutions visible on the board, together at the same time. This is the way Japanese teachers typically support their students' comparisons between solution strategies (Stigler and Hiebert 2004; Richland et al. 2007), and has been shown to improve learning (Begolli and Richland 2016). Interestingly, 44% (15 participants) of those who made these solutions visible simultaneously also used linking gestures between them suggesting pedagogical supports to make the comparison more salient to students.

When looking at the verbal comparisons that participants made between these solution strategies, preservice teachers most often made these comparisons themselves rather than eliciting "student" participation. Fifty-three percent (18 participants) of participants made statements about how the solution strategies were similar or different whereas only 26% (9 participants) asked the imaginary audience about how the solution strategies were similar or different. This finding may be related to the fact that this was a micro-teaching even and students were not actually sitting in front of the teachers. However, it is important to notice that that many of the participants elicited "student" talk by asking a question and then pausing theoretically for a response, but just not at the point of making the comparisons.

Not quite half of the preservice teachers (47% or 16 participants) verbalized a judgment about which solution was preferable despite one of them being incorrect; a slightly higher number (53%, 18 participants) gave a statement about how or why the verbalized or written solutions were similar or different. Twenty-six percent (9 participants) of participants asked the imaginary audience how the solutions are similar or different and 24% (8 participants) asked the imaginary audience which solution strategy is preferable. Participants in this sample did retain control of the comparisons—verbalizing the main judgments and similarity/difference statements—as supported by evidence from a prior analysis of U.S. teachers' practices in the everyday TIMSS lesson sample (Richland et al. 2004).

Table 3 Frequencies practices of supporting comparing contrasting cases for those who presented both solution strategies (n = 34)

Code	Frequency	Standard error	CI lower	CI upper
Participant writes the step-by-step reasoning/calculation incorrect solution	0.79	0.07	0.65	0.94
Participant writes the step-by-step reasoning/calculation correct solution	0.79	0.07	0.65	0.94
Participant explains reasoning/calculation processes to solve a new problem	0.15	0.06	0.02	0.27
Whole step-by-step reasoning for more than one solution on the board together	0.50	0.09	0.32	0.68
Participant moves hand directly from one solution to another	0.50	0.09	0.32	0.68
Participant asks class which solution they think is preferable	0.24	0.07	0.09	0.39
Participant gives judgment about which solution is preferable	0.47	0.09	0.29	0.65
Participant asks class how the solutions are similar or different	0.26	0.08	0.11	0.42
Participant gives statement about why or how the presented solutions are similar or different	0.53	0.09	0.35	0.71
Participant asks class how the presented solutions are different than a new problem	0.00	0.00	0.00	0.00
Participant presents an additional solution to problem and explains how they are similar or different	0.15	0.06	0.02	0.27

Use of cues to support comparisons for subset who presented a new problem

Observations of the data revealed that despite being provided with one problem and two solutions in the prompt, some participants invented a new problem to use as a comparison with the originally posed problem, perhaps to demonstrate the transfer process. While contrasting solution strategies was the focus of our study, the following analysis led to information about how participants made comparisons between problems. Thus, this third set of results shows the frequencies for the support strategies used by those twenty-two participants who presented a new problem to illustrate the concept of $y = 2x + 3$. Within this group 59% (13 participants) of the participants who invented a new problem verbalized the step-by-step reasoning process for the correct solution strategy (45%, 10 participants, wrote it down as well), whereas only 27% (6 participants) of participants verbalized the step-by-step reasoning process for the incorrect solution strategy (and 23%, 5 participants, wrote it down). Most of these participants explained the step-by-step reasoning process of the correct solution strategy and presented students with a new problem that illustrated the same use of that strategy. Forty-one percent (9 participants) of these participants also explained how their problem was similar or different to the given solution strategy; however, only one participant asked the imaginary audience to compare how the solution was different than the new problem, suggesting that these preservice teachers were less attuned to asking students to make these comparisons themselves and more attuned to making these comparisons for their students. We would like to remind the readers that because participants performed their lesson in front of an imaginary classroom the results presented here on participants eliciting student responses may in fact be under-estimates had students actually been present in the classroom. Nevertheless these results are in line with previous research suggesting that preservice teachers are not always oriented to

Table 4 Frequencies practices of supporting comparing contrasting cases for those who presented a new problem (n = 22)

Code	Frequency	Standard error	CI lower	CI upper
Participant goes through both of the solutions?	0.23	0.09	0.04	0.42
Participant verbalizes the step-by-step reasoning for the incorrect strategy?	0.27	0.10	0.07	0.47
Participant verbalizes the step-by-step reasoning for the correct solution strategy?	0.59	0.11	0.37	0.81
Participant writes the step-by-step reasoning/calculation misconception	0.23	0.09	0.04	0.42
Participant writes the step-by-step reasoning/calculation correct solution	0.45	0.11	0.37	0.81
Whole step-by-step reasoning for more than one solution on the board together	0.32	0.10	0.11	0.53
Participant asks class how the presented solutions are different than a new problem	0.05	0.05	−0.05	0.14
Participant presents an additional solution to problem and explains how they are similar or different	0.41	0.11	0.19	0.63

eliciting and responding to student responses (e.g., Santagata and Yeh 2014). There did seem to be a small group of participants who were highly attuned to comparisons, that is, those who presented a new problem as well as presented the two solution strategies given in the example (23% of this sample of twenty-two, or five of the original sample of 80) (Table 4).

CKT-M

Next, we examined whether individual differences in participants' engagement with the coded teaching strategies related to their performance on the knowledge for mathematics for teaching measure (CKT-M; n = 74). Participants averaged 17 items correct on the CKT-M out of 33, with standard deviation of 7.90, min of 0, and max of 33. The distribution of scores of the CKT-M was normal. We present point biserial correlations between the total number correct on the CKT-M and the total number of strategies coded for and include correlations from a median split of performance on the CKT-M to illuminate any potential differences between participants with high and low mathematics content knowledge (Table 1). A point biserial correlation between the total number correct on the CKT-M and the total number of strategies coded for was 0.18 ($p = 0.12$); however, when looking at results from the median split, we found participants in the high CKT-M group had a correlation of 0.80 ($p < 0.001$) between scores on the CKT-M and number of strategies. When looking at the correlation between the total number correct on the CKT-M and specific strategies for comparison, we found no significant correlations; however, results from the median split indicated that participants in the high CKT-M group were significantly more likely to verbalize both solution strategies ($r = 0.27$, $p = 0.02$) and verbalize the incorrect solution strategy ($r = 0.28$, $p = 0.02$). This suggests that the variability in use of comparisons was only partially explained by their knowledge of mathematics for teaching but their use of support strategies was not.

Discussion

Results from this study showed that preservice teachers in our sample, like practicing teachers observed in the TIMSS video studies, did not regularly plan and enact a discussion of student solutions within the micro-teaching event. Those who did, often did not provide visual support cues to highlight the key comparisons. The majority of participants only presented one student solution and therefore missed out on opportunities to engage students in contrasting a correct solution with an incorrect solution. This was also true for participants who chose to present the reasoning process to solve a new problem, thereby focusing on transfer. Most participants did not provide a comparison between problems, and if they did, the majority did not use support cues known to be important to ensuring successful learning. When looking at correlations between those who scored high on the measure of mathematical content knowledge and those who scored low, we found that mathematical content knowledge only partially explained whether participants used contrasting cases in their lesson but not whether they used pedagogical cues to support those contrasting cases. The results are surprising given that a mathematical content knowledge should enable teacher to engage in instructional practices that highlight more foundational structures of concepts. As such, the reasons behind these patterns are not clear. We discuss four potential explanations for these data as well as future directions and limitations of this work. We acknowledge that there may, of course, be additional contributing factors that influence a teacher's practice. We ultimately suggest that preservice teachers are simply not oriented to using contrasting cases in teaching, so any professional development designed to support leading classroom discussions comparing solution strategies would have to begin by demonstrating the importance of the practice.

First, it is possible that teaching practices of comparison are related to the teachers' mathematics knowledge in that more systematic, conceptual knowledge of mathematics would lead teachers to more explicitly represent the deep structure of mathematical representations (Ma 1999). Importantly, however, we found only partial evidence of an association between mathematics for teaching knowledge and effective teaching strategies suggesting that mathematical knowledge for teaching may only partially explain our findings. Some of the participants in our sample expected to be teaching mathematics with their credential (either at the elementary or high school level), but we expected their knowledge of mathematics for teaching scores to be more related to their teaching practices than the type of credential they were working toward. It is possible, however, that type of credential could be an important predictor and we therefore urge future research to take this into account.

A second possibility is that practicing teachers intend to support their students in comparing solutions, but do not execute these practices due to contextual factors that are not related to the actual teaching of mathematics such as the physical layout of the room, time management, and transitions between tasks (e.g., Henningsen and Stein 1997). Henningsen and Stein (1997) found that classroom management problems contributed to teachers focusing on teaching mathematical procedures without an emphasis on making connections and teachers doing activities in class that are not related to mathematics. Worse yet, classroom management problems can even play a large role when "tasks decline into complete lack of mathematical engagement on the part of the students" (p.537). Because the teaching lesson was done in a controlled environment, however, we expect that the micro-teaching event reflected these teacher candidates' best intended teaching practices, and their participant's lack of effectively using contrasting cases is not explained by simultaneous classroom management goals. Information from participants' lesson plans—which were not collected as part of this study—could be used to better understand whether preservice teachers intend to use contrasting cases and pedagogical supports.

A third possibility could be that the participants did not focus on the incorrect solutions because mistake-handling can cause anxiety for teachers In particular, these results may reflect orientations to the use of incorrect strategies, since these teacher candidates were least likely to present the incorrect strategy. The low rates of presenting both solution strategies (under 54%) may be partially explained by the culture surrounding mistake making and incorrect answers in the United States. While teachers must confront incorrect student solutions, the handling of these mistakes is likely to cause anxiety in the student (Santagata 2004). In a study comparing mistake making in the United States and Italy, Santagata (2004) found that in the U.S., making mistakes was handled privately between the teacher and the student instead of when the student was at the board. This hesitation to make mistakes public may influence how teachers present incorrect solutions to students and may make them hesitant to engage in comparison strategies with the student misconception. In addition, our preservice teachers may have been hesitant about presenting incorrect answers on the board because they think that students may recognize the incorrect strategy as the correct one and thereby engage in negative learning (Dalehefte et al. 2012). Teachers may not be attuned to the fact that errors and mistakes in math classrooms can serve as a useful learning tool for students (Santagata 2005).

At the same time, concern about public discussion of a student mistake might explain the low rate of comparisons between the two solution strategies, but an even smaller percent of the sample made a comparison between the solved problem and a new problem. Thus overall, we suggest that beyond the specific case of comparisons between a misconception and a correct solution, these preservice teachers were not oriented toward noticing the utility of making a comparison within their approximation of practice. Implications of this study suggest that deliberate instruction would be necessary to orient preservice teachers around using contrasting cases in their instruction. A study by Bartell et al. (2013) found that a video-enhanced intervention in which teachers were asked to examine many examples of eliciting student thinking, lead to increases in preservice teachers' ability to analyze conceptual responses to students. Though the study did not investigate whether the intervention actually changed instructional practices implications suggest that instruction could be designed to specifically orient preservice teachers around the use and scaffolding of contrasting cases. Other studies demonstrate that preservice teachers can be taught to attend to specific aspects of instruction but that it takes deliberate effort to do so (e.g., Santagata and Yeh 2014; Stockero 2008). Future research should investigate whether such interventions lead to changes in teacher practice.

Finally, it could be that even though in participants may have been exposed to ideas about contrasting cases in previous coursework, participants only learned about them in a theoretically focused way but not for practice. Much research highlights the gap between theory and practice in teaching (e.g., Korthagen and Kessels 1999; Lampert 1990; McDonald et al. 2014) and it may be that although preservice teachers have exposure to ideas around comparing contrasting cases, they are not attuned to explicitly using this knowledge in their practice. We suggest that a more practice-based approach (i.e., McDonald et al. 2014) to teacher preparation could allow future teachers to develop practices that are tied to underlying theories of learning. Most of this work locates "practice-based pedagogies" in methods courses but the findings suggest that this pedagogy could benefit candidates program-wide.

Limitations

Several limitations to this study need to be mentioned. First, preservice teachers were videotaped in a short (15-min) simulated micro-teaching event rather than a real classroom setting. This was done purposefully to understand participants' planned intentions for how they would teach these materials, and to minimize variability between teachers' classroom contexts, which would understandably change their teaching practices. Further, the controlled setting allowed us to give all participants exactly the same task in a more controlled setting—something not necessarily feasible in a real classroom setting. Even so, the approximated nature of the classrooms may mean that the results may not fully represent the classroom-based teaching strategies that would have been employed by these teachers, though the high consistency with teaching practices identified in the TIMSS studies suggests these may not be entirely different. It could also be that 15 min is too short a time for participants to teach the concept and engage in a discussion with the imaginary audience suggesting that participants may have had the intention of engaging in a class discussion, but due to the time constraints of the task could not. Further studies could collect lesson plans or interview participants on their intentions of practice to understand this issue.

Second, results presented in this paper focus on a special case of comparison between two student solutions. While this is a highly important and under-used strategy in everyday teaching, it may be that preservice teachers are more attuned to make comparisons between two problems than two solutions to the same problem. While we saw some evidence for this in our teachers who invented new problems, we cannot speak to this since our prompt drew attention to solution strategies. Thus, our conclusions regarding teachers' orientations toward making comparisons in classrooms most specifically explain teachers' orientations toward the use of students' solution strategies in comparisons. It could be that further coding of participants' practices that were not part of the existing coding scheme could illuminate new strategies that were not anticipated by the literature.

Third, the sample included a diverse sample of preservice teachers; some were receiving a certification to teach all content areas in the early grades while others were studying to teach mathematics in the middle and high school years. We were not able to examine these groups separately, so there may be differences in the orientations of these candidate groups. At the same time, we expected that we would have seen key differences reflected in their CKM-T results, which only partially explained these performance differences in the results of the median split of the sample. In fact all of our candidates displayed fairly similar (low) performance, which is not uncommon (e.g., Kajander 2007; Newton 2008; Seaman and Szydlik 2007). Future research should more closely investigate differences between participants receiving a primary certification and those receiving a secondary certification.

Conclusion

In sum, the sample of preservice teachers showed infrequent use of comparisons between student solutions in a micro-teaching event, and many of those who did make comparisons used few visual strategies to support their students in making those comparisons. This suggests that teacher candidates are not orienting to comparison as an effective practice, and thus creating learning experiences within teacher preparation programs may help preservice teachers better engage in practices of comparing and contrasting. Particularly, a more practice-based approach that helps preservice teachers more effectively translate theory into practice would be most promising. Encouraging classroom teaching with comparing and contrasting solutions may require a conceptual shift on the part of teacher candidates, rather than simply including mention of this pedagogy in credentialing programs or curriculum guides (as it has been recently added into the United States Common Core Curriculum). Curriculum reform is increasingly emphasizing these practices, and therefore more explicit attention to this apparent misalignment between teacher orientations and practices is important to consider when designing learning experiences for these preservice teachers.

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