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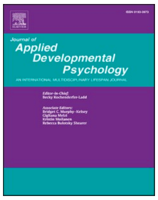

Buckets of fun: Impacts of fraction ball activities on students' math-related emotions

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ABSTRACT

Rational number learning can cause frustration and negative emotions for elementary school students. Fraction Ball, a play-based math intervention, allows students to actively learn rational numbers through engaging and interactive activities. Based on a cluster-randomized trial with 16 teachers and 360 students, our pre-registered analyses showed moderate positive impacts of Fraction Ball on overall students' self-reported math-related emotions. Exploratory analyses indicated that students with higher negative emotions at pretest showed larger experimental impacts on decreasing negative emotions at posttest. Finally, we found that Fraction Ball evidenced no trade-off between rational number learning and emotional outcomes at the classroom block level, indicating that positive learning gains in rational number skills were associated with increases in positive emotions and decreases in negative emotions.

Negative emotions about math, such as frustration and anxiety, are commonly experienced by young learners in elementary schools (Aarnos & Perkkilä, 2012; Ashcraft & Ridley, 2005), and they have been found to be more strongly related to student performance when dealing with demanding mathematical concepts such as fractions compared to less complicated problems (Ashcraft & Ridley, 2005; Starling-Alves, Wronski, & Hubbard, 2022; Wu, Barth, Amin, Malcarne, & Menon, 2012). Moreover, despite the initial increase in math self-concept at the age of five to six, students' self-evaluations of mathematics abilities tend to show the largest decreases at the age of 10–12 (Orth, Dapp, Erol, Krauss, & Luciano, 2021), which corresponds with the introduction of rational number concepts into the curriculum. According to the Situated Expectancy-Value Theory (SEVT), students' emotional responses to math contribute to shaping their subjective value of math learning, potentially influencing their effort and persistence in math-related activities (Eccles & Wigfield, 2020; Jiang, Rosenzweig, & Gaspard, 2018; Safavian, 2019). Because mathematics is a cumulative academic field, enhancing elementary students' emotional experiences in math education can plausibly increase their future participation in math-related career trajectories.

Play-based learning has shown promise in improving achievement, mitigating negative emotional experiences, and might address known challenges of learning rational number concepts (Parker, Thomsen, & Berry, 2022). Indeed, our previous research has found that a playful math intervention—Fraction Ball—improved students' overall rational number learning in a cluster-randomized experiment with 360 fourth- and fifth-grade students (Begolli et al., 2023; See Fig. 1 for a brief description of the Fraction Ball game; See Bustamante, Begolli, Alvarez-Vargas, Bailey, & Richland, 2022 for a full description of Fraction Ball pilot design and development). However, popular arguments and recent studies have raised concerns about whether effective math learning opportunities might sometimes conflict with social-emotional math-related outcomes, such that truly effective mathematics achievement gains may require educational practices that cause discomfort due to being challenging (e.g., Blazar & Pollard, 2023; Boaler, 2014). The purpose of the current study was to test whether, in a game-based

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context, students' emotional outcomes aligned or were counter to their positive math learning gains after the Fraction Ball intervention.

Although negative math-related emotions and math achievement are consistently found to be negatively associated (e.g., Barroso et al., 2021), scholars have raised concerns that some effective math activities may increase negative emotions at the same time as they raise math learning. For example, Boaler (2014) finds that based on student reflections about mathematical experiences, this may be the case for timed math tests. Consistent with this concern, recent work combining student test scores with videos of classroom activities has found that activities with higher student learning are frequently associated with lower student happiness and enjoyment with mathematics and vice versa (Blazar & Pollard, 2023). These findings do not have clear implications for classroom practice: first, the evidence for the effects of speed pressure, such as speeded practice (e.g., practicing or being tested on math problems under time pressure) and timed tests on student math-related emotions is mixed (Caviola, Carey, Mammarella, & Szucs, 2017; Namkung, Peng, & Lin, 2019). Further, given the apparent benefits of improvements in students' math achievement for reducing their later math anxiety (Gunderson, Park, Maloney, Beilock, & Levine, 2018; Ma & Xu, 2004; Wang, Rimfeld, Shakeshaft, Schofield, & Malanchini, 2020), it is possible that this potential trade-off can be circumvented if practice leads to learning improvements. Consistent with this hypothesis, the SEVT posits that individuals' perceived emotional costs of engaging in a learning task typically involve the anticipated frustration and anxiety from failure (Eccles & Wigfield, 2020). Fraction Ball's repeated opportunities to add fractions, convert between fractions and decimals, and locate these numbers on the number line in collaboration with peers may lead to conceptual understanding at the same time as fluency. This greater competency, in turn, could increase students' expectancies for later success and minimize their perceived emotional cost for future activities involving rational numbers, thereby improving their math-related emotions by effectively growing their math skills. This aim to produce learning with understanding maps onto the goals for learning detailed by the National Council of Teachers of Mathematics Standards (NCTM, 2000) and Common Core State Standards (Common Core State Standards Initiative, 2010), such that meeting standards for learning with understanding should improve math attitudes and facilitate future learning.

A second mechanism through which Fraction Ball might alter students' math-related emotions is through its playful activities. One exception to the pattern of apparent trade-offs between activities that promote math learning and math engagement was active mathematics that involved hands-on participation, physical movement, and social interaction, which were associated with higher gains on both test scores and student self-reported enjoyment (Blazar & Pollard, 2023). These active mathematics elements are all evident in Fraction Ball's play-based activities. Play is theorized to produce positive emotions (Kuczaj & Horback, 2012), and play in math has been demonstrated to increase learning (Ramani & Siegler, 2008). Importantly, while the word "play" colloquially suggests fun and positive emotions (e.g., happiness), the impacts of play-based math activities on both students' math learning in relation to math-related emotional outcomes haven't been assessed directly. The developmental literature on play is underdeveloped in regard to affect, and so the positive valence of math games, in particular, is not well understood. Playful math games such as Fraction Ball could conceivably lead to negative math-related emotions due to the games' competitive and potentially high-pressure nature. Conversely, it is possible that play-based interventions are fun and increase positive emotions, but the level of enjoyment may come at the expense of learning, creating a difficult trade-off for educators to navigate. Given that Fraction Ball activities consist of both speeded practice and play-based mathematics, our intervention provides a unique context to test whether active and playful math environments can circumvent the posited trade-offs between learning and emotional outcomes in mathematics.

Further, we explored whether classrooms that produced larger effects on learning outcomes also produced larger effects on math-related emotions. Negative impacts on math-related emotions, and/or a negative trade-off between impacts on math skills and math-related emotions across classrooms, would raise concerns that these activities improve students' math skills at the cost of introducing negative math-related emotions. Positive impacts on math-related emotions, with a positive association between impacts on math skills and math-related emotions, would indicate that, at least in the context of Fraction Ball, improvements in math learning do not compromise improvements in math-related emotions. Moreover, while we cannot distinguish between whether a positive relationship between learning gains and achievement would derive from the effective mathematical practices students participate in during Fraction Ball or from enjoyment raised by the playful nature of the games, finding a positive relationship between math outcomes and positive emotions, and a negative relationship between math outcomes and negative emotions, would provide important insight into whether math interventions that create enjoyment must trade-off with achievement. We'll situate this discussion into the literature on emotion, mathematics achievement, and play more fully below.

(a) (b)

Fig. 1. Pictures of students playing fraction ball games on the redesigned school basketball court.
Picture (a) shows students engaging in Fraction Ball games on a redesigned school basketball court that reflects fractions (represented by the green area) and decimals (represented by the blue area). In addition, as shown in (b), the court features a life-size number line on the side. During Fraction Ball games, students form teams and rotate through different roles such as shooter, rebounder, and counter. For instance, when a shooter successfully scores a shot worth 1/4, the rebounder announces the corresponding value, "1/4," and the counter moves forward on the number line, adding 1/4 to their team's score.

Math-related emotions

Students' affective experiences are crucial components of learning and instruction generally (Kaplan & Garner, 2017), and specifically in mathematics education (Evans, 2000). Students' emotional reactions to mathematics, such as feelings of happiness or frustration when solving math problems, are one of the three main constructs of the affective domain in math education, along with beliefs and attitudes (McLeod, 1992). Nevertheless, research on students' emotional experiences while learning math remains insufficiently developed, partly because of a lack of theoretical frameworks that explicitly articulate the role of emotions in math learning, as noted by Lewis (2013) and McLeod (1992). NCTM has encouraged educators to prioritize students' affective experiences in math, nurturing students' confidence, interests, curiosity, and persistence during math activities (NCTM, 1989, 2014). Following the NCTM and Common Core State Standards, the recent Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve has also emphasized the importance of meaningful and engaging math learning that facilitates the development of positive attitudes toward math (California Department of Education;, 2023).

Negative emotions and anxiety about mathematics have received particularly close attention when considering the relations between affect and math (e.g., McLeod, 1992). The prevalence of negative emotions about math in student populations has been well documented worldwide (see Lau, Hawes, Tremblay, & Ansari, 2022), and their relations to achievement are varied, though generally more negative math-related emotions are related to negative achievement patterns (e.g., Ashcraft & Moore, 2009; Richardson & Suinn, 1972; Vukovic, Kieffer, Bailey, & Harari, 2013). Some scholars have described negative math-related emotions as a major obstacle to math learning and performance (Li, Chen, & Zhou, 2023). Among young learners, fractions are one of the most crucial but challenging math topics, and studies have shown that fraction learning is closely related to math anxiety (Li et al., 2023; Starling-Alves et al., 2022). Negative math-related emotions (e.g., frustration and anxiety) might affect students' motivation to avoid fraction learning activities.

Positive emotions among students in math education have not received equal focus (Pinxten, Marsh, De Fraine, Van Den Noortgate, & Van Damme, 2014; Villavicencio & Bernardo, 2016). According to the broaden-and-build theory (Fredrickson, 2003), positive emotions, in general, can lead to gains in various developmental domains, including physical (health and strength), cognitive (problem-solving and learning), psychological (identity, goal orientation, and resilience), and social (relationship building) aspects. In particular, Fredrickson (2001) highlighted the role of play, which was believed to be closely associated with one positive emotion–joy, in leading to those improvements in different domains. In mathematics specifically, previous research has found that elementary and middle school students' prior enjoyment of math was positively related to their subsequent math competence beliefs and perceived learning effort (Pinxten et al., 2014). Moreover, positive academic emotions (enjoyment and pride) were also found to predict math achievement beyond anxiety (Villavicencio & Bernardo, 2016). In addition, Karamarkovich and Rutherford (2021) developed an emotion measure rooted in the control-value theory of academic emotions (Pekrun, 2006), which places greater emphasis on positive emotions than negative emotions (Villavicencio & Bernardo, 2016). They identified four meaningful profiles of math-related emotions—two positive profiles (one distinct emotion being hope in one profile and its absence in the other), one negative profile, and one mixed profile. They also found these emotion profiles mediated the relationship between math expectancy/value and achievement (Karamarkovich & Rutherford, 2021).

The Control-Value theory of academic emotions, which focuses on emotions related to achievement activities or outcomes, suggests that individuals' control appraisals (e.g., expectations and attributions) and value appraisals (e.g., intrinsic and extrinsic values) of the activities and performance outcomes are the most important determinants of their emotions (Pekrun, Frenzel, Goetz, & Perry, 2007). In addition to the valence of emotions (positive vs. negative), the theory introduces a three-dimensional taxonomy of different academic emotions, prompting attention to the other two dimensions: activity vs. outcome focus and the degree of activation (activating vs. deactivating). Considering the potential impact of Fraction Ball on altering students' subjective controls and values toward math learning (as described in the Improved Rational Number Skills subsection), we followed Karamarkovich and Rutherford (2021) to evaluate the intervention impacts on the emotions grounded in the Control-Value theory. The emotion measure also covers different emotion categories, as classified by Pekrun et al. (2007). For instance, hope represents a positive, potentially activating emotion, while boredom reflects a negative, potentially deactivating emotion. In the current study, we examined the impacts of the intervention on both discrete emotions (i.e., the seven emotions outlined in Karamarkovich & Rutherford, 2021) as well as positive and negative emotion composites. Although students tend to exhibit different emotional profiles related to math, as reported by Karamarkovich and Rutherford (2021), previous studies did not yield specific hypotheses regarding the potential varied effects of Fraction Ball on different emotions. Therefore, we present the impacts on both positive and negative emotion composites and individual emotion items, which provides a full picture of the intervention effects and allows for comparison across impacts on different emotions.

Complicating the difficult task of improving students' math-related emotions is the concern that optimizing instruction to improve their math skills alone may backfire. Despite the consistently positive linkages between students' emotions and achievement, which were evident in elementary school (Lichtenfeld, Pekrun, Stupnisky, Reiss, & Murayama, 2012), studies focusing on the impact of teachers on different types of student outcomes (e.g., cognitive vs. social-emotional) have found that the effects of these different types of teacher practices on math achievement and engagement were only weakly or even negatively correlated with each other (Blazar & Kraft, 2017; Blazar & Pollard, 2023; Kraft, 2019).

Specific practices of interest in the current study are the use of timed math tests and speeded practice, both of which were integral elements of the Fraction Ball program. As a part of pre-and post-intervention rational number knowledge tests, all student participants completed a three-minute timed test. In addition, in several Fraction Ball games, students practiced adding fractions under time pressure. Timed math tests, where students are informed about an explicit and limited time to finish math tasks (Grays, Rhymer, & Swartzmiller, 2017), are commonly used in math instruction. Timed testing has been shown to be an effective practice for building fluency (e.g., Rhymer et al., 2002). Similarly, speeded activities are typically designed to foster automaticity with central math problem-solving strategies (Fuchs et al., 2014). However, many scholars have expressed concerns that speed pressure could aggravate students' math anxiety (Boaler, 2014, 2015; Geist, 2010). Nevertheless, a recent meta-analysis pointed out that due to the limited number of studies, the causal links between timed tests and math anxiety and performance lack evidence (Namkung et al., 2019). For these reasons, more investigation into the relationship between learning context and emotional valence is warranted. We aim to do so by exploring whether or not using play-based and active learning approaches could mitigate the potential trade-offs between timed drills and student social-emotional outcomes in math education.

Fraction ball as a playful and active mathematics approach to promote positive emotions

The current study uses a promising playful learning model that integrates play with math learning opportunities designed with the science of learning principles (Hirsh-Pasek et al., 2015), briefly reviewed below. Fraction Ball play activities are combined with in-class instructional periods, with learning tested with speeded and non-speeded assessments. We discuss in more detail the theory behind two potential pathways through which Fraction Ball might influence students' math-related emotions: affordances of play-based learning for positive emotional experiences and improved rational number skills.

Affordances of play-based learning activities

Play is theorized to be an important contributor to children's emotional development (Mellou, 1994; Verenikina, Harris, & Lysaght, 2003). Early theorists speculated about the psychological benefits of play, suggesting that play provides a safe environment for children to express their negative emotions and may help buffer the effects of negative feelings (Erikson, 1963; Freud, 1961). More recently, social-cognitive benefits have been proposed; for example, Kwon and Yawkey (2000) suggested that children learn to express and regulate their emotions through play and that play provides a responsive environment for adults to provide positive models for children to learn emotional expressions. In addition, play has consistently been recognized for its positive affect and pleasing nature, as well as its high correlation with positive emotions (Kuczaj & Horback, 2012). In particular, from the perspective of positive psychology, the positive emotions (e.g., joy) elicited during play can increase resources for children's cognitive (e.g., attention and thinking) and social (e.g., social bonds) development (Fredrickson, 2004).

Given play can help children's adjustment and enhance both their school readiness and social-emotional development, researchers argue that play should be an integral part of children's school settings (Ginsburg, the Committee on Communications, & the Committee on Psychosocial Aspects of Child and Family Health, 2007). Playful learning is an example of this educational approach, exposing children to academic content through engaging activities promoting whole-child development (Hirsh-Pasek, Golinkoff, Berk, & Singer, 2009). Math education could benefit from playful learning as students tend to exhibit various attitudes toward this particular subject (Zosh, Hassinger-Das, Spiewak Toub, Hirsh-Pasek, & Golinkoff, 2016). Indeed, fundamental math concepts (e.g., counting and estimation) are already evident in children's free play, and these early math skills can be further developed in an enjoyable way via playful learning (Ginsburg, 2006). In playful math learning literature, game play has been the most promising approach (Ilgaz, Hassinger-Das, Hirsh-Pasek, & Golinkoff, 2018). Math games have a high potential to benefit student learning, motivation, engagement, and enjoyment (e.g., Monroe & Nelson, 2003; Ramani, Daubert, & Scalise, 2019; Russo, Russo, & Bragg, 2018). Specifically, games with explicit learning goals provide opportunities for students to actively apply their math skills to solve problems or reach goals, collaborate with peers with different math abilities, and ultimately help and learn from each other (Ramani et al., 2019). These insights from the math game approaches were incorporated when designing the Fraction Ball activities discussed in this study (Alvarez-Vargas et al., 2023). Students formed teams to play interactive basketball games, using their rational number knowledge to keep and compare scores in order to achieve a meaningful goal (e.g., winning the game).

Additionally, the various physical movements involved in playing Fraction Ball can potentially enhance students' emotional experiences, as research suggests that physical activity plays a positive role in boosting mood (Bustamante, Santiago-Rodríguez, & Ramer, 2023; Stillman, Cohen, Lehman, & Erickson, 2016). Additionally, engaging in physical activity has also been linked to the reduction of negative affect and improvement in cognitive functioning among children (Bidzan-Bluma & Lipowska, 2018; Lubans et al., 2016). Indeed, Fraction Ball—which involves shooting a ball, running on the court, and moving on the number line—engaged students in a higher level of physical activity than traditional classroom math activities (Bustamante et al., 2022), which in turn can improve students' math-related emotions.

Furthermore, the games also gave opportunities for teachers to provide scaffolding and feedback when students were using fractions to calculate scores or marking fractions on the number line painted on the court. For example, after students finished a Fraction Ball game, teachers asked, "How many points do you need to equal the score?" and were able to provide feedback to the whole group after they answered and worked through potential misconceptions that may have produced incorrect solutions. As such, feedback was embedded in the student-teacher discussions, with teachers having the freedom to provide feedback to students as they saw fit. In addition, each team's tracker and checker roles are designed to provide a peer feedback loop because they are responsible for ensuring the points are marked and added correctly along the number line. These roles also extend to group work in the classroom. Further, materials are designed such that students can check their answers between different representations (e.g., using a number line for addition versus a number sentence), giving them more chances to identify incorrect solutions during the activities. Such support from teachers and peers can reinforce learning and address potential misconceptions that students may have in a low-stress, playful environment; it also has been shown to increase students' motivation (Harks, Rakoczy, Hattie, Besser, & Klieme, 2014). Moreover, this type of support might be especially helpful for our target population of low-income and Latine students who participated in this study, who belong to groups traditionally marginalized in math education and are likely to have more negative previous experiences with math (Copur-Gencturk, Cimpian, Lubienski, & Thacker, 2020; MacPhee, Farro, & Canetto, 2013).

Fraction Ball's design also anticipates concerns about trade-offs between optimizing for math learning and social-emotional outcomes. In contrast to emotional states (and teaching and learning culture) found in traditional classrooms (e.g., Lewis, 2013), such as students experiencing anxiety for not wanting to have the wrong answer, the game environment may provide affordances for students that normalize trial and error and motivate students to continue to iterate with the math materials. In math classrooms, Blazar and Pollard (2023) reported that active math activities featuring hands-on participation, game engagement, peer interaction, student choice, and physical movement were associated with improved math achievement and self-reported engagement and happiness levels. Moreover, despite the fact that Fraction Ball consisted of timed portions in pretest and posttest and speeded practices during the games, students were informed that these tests and game performances would not affect their school grades. For these reasons, the playful, active, low-stress, and interactive features of Fraction Ball were theorized to have the potential to mitigate the potential trade-offs between learning and emotions described in the *Math-Related Emotions* section of the Introduction.

However, despite the potential benefits of playful math activities, there has been limited empirical and particularly experimental evidence to support their role in promoting students' math-related emotions. While some studies have shown that digital games might have the potential to alleviate math anxiety (e.g., Ng, Chen, Wu, & Chang, 2022) and enhance fraction attitudes (e.g., Riconscente, 2013), most of the math-related playful learning literature focuses on cognitive outcomes, such as math thinking or learning (e.g., Ramani & Siegler, 2008), as opposed to social-emotional outcomes. However, effective math activities that enhance math achievement can sometimes decrease students' enjoyment of math learning (Blazar & Pollard, 2023). In fact, Bragg (2007) found conflicting results between survey and interview data on the relationships between math games and students' positive attitudes. Therefore, by investigating the impact of Fraction Ball games on students' math-related emotions, this study can contribute to our understanding of the potential of playful learning to impact emotional experiences in math learning. We also attempt to investigate the potential for a math learning vs. math-related emotions trade-off in implementations of Fraction Ball by testing whether classrooms where Fraction Ball had the largest impacts on rational number skills also had smaller impacts on math-related emotions (indicating a potential trade-off) or larger impacts on math-related emotions (indicating that emotion and cognition outcomes moved together).

Improved rational number skills

Fraction Ball may also enhance students' math-related emotions by improving their rational number skills. According to the SEVT, individuals' perceived emotional or psychological costs of engaging with a learning task typically involve the anticipated frustration and anxiety from failure (Eccles & Wigfield, 2020). Based on this theory, Fraction Ball might help reduce students' perceived emotional cost of rational number learning in a playful and active setting, thereby improving students' subjective task values. More specifically, in Fraction Ball, students effectively improved their rational number knowledge in a low-stress, playful learning environment (Begolli et al., 2023; Bustamante et al., 2022). This improvement might help students decrease their perception that learning fractions and decimals is too emotionally costly. As a result, students may be less likely to feel sad, frustrated, or stressed when taking the rational number knowledge test after the intervention.

In sum, this study evaluated whether the Fraction Ball intervention that leveraged the science of learning principles provided a positive emotional experience while learning complex math concepts (i.e., fractions and decimals). There has been an increasing recognition of the need to integrate the science of learning principles, which are built on several intertwined fields of research, into classroom instruction (Cantor, Osher, Berg, Steyer, & Rose, 2019; Darling-Hammond, Flook, Cook-Harvey, Barron, & Osher, 2020; Dunlosky, Rawson, Marsh, Nathan, & Willingham, 2013). Therefore, this study also contributes to the growing literature connecting research and practice as well as evaluating evidence-based practices in educational settings.

The current study

This study aims to explore the impact of the Fraction Ball intervention on students' math-related emotions assessed immediately after taking a low-stakes fraction test, and the associations between math learning and social-emotional outcomes. Specifically, the study addresses the following research questions:

RQ1: What are students' average self-reported math-related emotion levels toward math tests after the Fraction Ball intervention?

RQ2: Does the Fraction Ball intervention improve students' self-reported positive math-related emotions and reduce negative math-related emotions relative to "business-as-usual" math instruction and physical education (PE)?

RQ3: Are the impacts of the intervention on students' math-related emotions moderated by students' prior negative math-related emotions, grade level, and gender?

RQ4: What are the associations between impacts on rational number learning and math-related emotion outcomes at the classroom block level? For example, are classrooms with better rational number learning also demonstrating more gains in positive math-related emotions?

RQ1, RQ2, and RQ3 were listed as secondary ("exploratory") research questions in the pre-registration (https://osf.io/rjdpk). However, we preregistered hypotheses for RQ1 and RQ2, which were thus not fully exploratory in nature. For RQ1, our pre-registered hypothesis was that students' self-reported feelings of happiness at posttest would range from three to five (on a scale of one to five; one being *not at all happy*, five being *very happy*; see the *Measures* section of the Method) and their levels of feeling nervous would range from one to two (on a scale of one to five; one being *not at all nervous*, five being *very nervous*). For RQ2, we hypothesized that Fraction Ball would decrease students' negative math-related emotion composite scores and nervousness as well as increase their positive math-related emotion composite scores and happiness.

We had no pre-registered hypotheses about RQ3 and RQ4. We view RQ3 as an exploratory question. We were interested in examining the potential moderating roles of pretest negative emotions, grade level, and gender to explore the extent to which our intervention may evidence differential treatment impacts for subgroups of students. Understanding which students the intervention is (or is not) working well for could inform iterations of the program. However, reasonable arguments could be made from theory and our previous results with this program to support predictions of impacts in both directions for these moderators. Specifically, for the pretest negative emotions, on one hand, students with lower levels of negative math-related emotions at baseline might have a more positive emotional experience during Fraction Ball because they enjoy math; on the other hand, the affordances of Fraction Ball, such as a low-stress environment and improvements in math learning, could be particularly helpful for students with higher pretest negative math-related emotion levels.

In terms of the moderating role of gender, it is possible that if students perceive basketball as a gendered activity, the intervention might disproportionately benefit boys' math-related emotions. However, in our design, we intentionally incorporated activities to engage girls, including activities in which students watched and analyzed short clips from the Women's National Basketball Association games. Further, our previous findings about the moderating effects of gender on math learning were mixed, in one implementation showing nearly identical learning gains by gender (Bustamante et al., 2022) and in another implementation showing larger impacts for boys than girls (Begolli et al., 2023;). Therefore, we did not have a strong prior hypothesis about differential impacts on math-related emotions between boys and girls. Lastly, past iterations of Fraction Ball did not yield significant differences in math learning outcomes across grade levels (Begolli et al., 2023; Bustamante et al., 2022). Thus, we had very little reason to believe the math-related emotions of students from different grade levels might be impacted differently. Therefore, despite our lack of strong a priori hypotheses, these exploratory research questions warranted examination as the results may help inform our program iterations and lead to a better understanding of the intervention's theory of action.

RQ4 is an exploratory analysis, which is not pre-registered. However, as described in the previous sections, our hope is that the learning and emotional outcomes would not trade-off in the context of Fraction Ball, which provides a playful and low-stress math learning environment.

Method

Fraction ball intervention

The data for the current study came from a cluster-randomized control trial of a math intervention called Fraction Ball. Fraction Ball provides an embodied, playful learning experience by redesigning school basketball courts to emphasize fraction and decimal learning (see Fig. 1). The traditional three-point arc on the court was converted to one point. Smaller arcs closer to the basket were assigned values of 1/4, 2/4, and 3/4 points on one end of the court and 1/3 and 2/3 points on the other end. Each hoop has fractions on one side and decimals on the other side. In addition, a walkable number line was painted on each side of the court to help students keep track of their scores.

Our Fraction Ball intervention consisted of six court activities and six classroom activities. See Bustamante et al. (2022) for a detailed description of the original court activities and Begolli et al. (2023) for the co-design process of the new classroom activities and revised court activities with 20 teachers. The six court activities allow students to practice estimating and comparing the magnitudes of fraction and decimal numbers. The six classroom activities are designed to analyze the strategies and performance during the games and review related fraction concepts (e.g., denominators and mixed numbers).

Participants

In this study, we partnered with 16 teachers and 360 students from four schools in the Santa Ana Unified School District in California. The study included ten fourth-grade classrooms with 208 students, five fifth-grade classrooms with 107 fifth-grade students, and a class with a mixture of fourth ($n = 10$) and fifth-grade students ($n = 35$). 97% of students were Latine, of which 83% of them qualified for free or reduced lunch, 47% were English Language Learners, and 52% were female students. This study was approved by the university's institutional review board (IRB).

Design

The experiment used a cluster-randomized design, such that we divided the 16 classrooms into eight blocks, matched based on classroom-level student pretest composite scores of rational number skills. For example, the two classrooms with the highest scores were in the first block, and the two with the lowest were in the eighth block. Then, one classroom in each block was randomly assigned to the intervention group, and the other was assigned to the control group. As a result, eight classrooms ($n_{students} = 198$) participated in the Fraction Ball intervention during math and PE time, and eight classrooms kept math and PE as usual ($n_{students} = 162$). Each school had at least one teacher in the intervention group and one in the control group. Therefore, we were able to conceptually treat these eight blocks of classrooms as eight different sub-experiments and explore whether sub-experiments that produced larger impacts on math learning produce larger, smaller, or unrelated impacts on math-related emotions.

Procedure

A one-class period pretest was administered two to three weeks before the start of the intervention. After the rational number items on the pretest, students' math-related emotions were assessed on the Tomoji measure (described below). We provided classroom and court lesson materials and a 90-min professional development session for the intervention group teachers. Six teachers completed all 12 activities, which included six classroom lessons and six court games. Two teachers completed eight out of the 12 activities. Finally, students in the intervention group completed the posttest within one week of the last day of activities. After the math items on the rational number posttest, students' math-related emotions were again assessed on the Tomoji measure. Students in the control group were administered the posttest at a similar time to the intervention group students in their block. See Begolli et al. (2023) for more detail on previously reported intervention procedures.

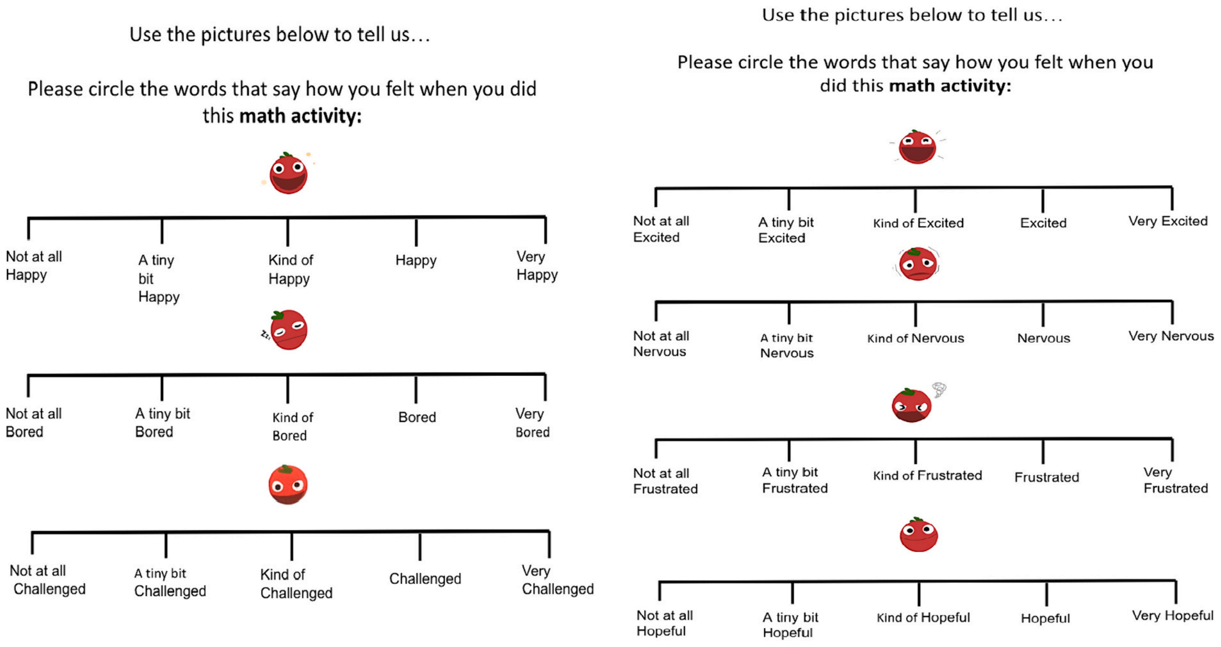
Fidelity of implementation

Trained observers from the research team conducted fidelity observations of 25% of the intervention sessions by recording the duration of each classroom/court lesson and marking the completion rate of the activities. Among the eight teachers in the intervention group, four completed all 12 activities, and two completed eight activities. The intervention was implemented across eight to fifteen class periods over three to six weeks. Teacher fidelity of the classroom lessons ranged from 41% to 86% and 37% to 82% for the court lessons.

Measures

The Tomoji measure of math-related emotions

Seven items adapted from Karamarkovich and Rutherford (2021) were used to measure students' math-related emotions during the pretest and posttest. These items were the final part of the pretest and posttest and were located after all the math problems in the tests. Students were asked to "circle the words that say how you felt when you did this math activity" for the seven items. Each item measured students' responses to a different emotion: happy, bored, challenged, excited, nervous, frustrated, and hopeful (listed in the order that was presented to students in the test; see Fig. 2). In order to help children understand the emotions being measured, a tomato emoji ("tomoji") was assigned to each item. We included the following options beneath each tomoji: Not at all [emotion], a tiny bit [emotion], kind of [emotion], [emotion], and very [emotion]. The responses were scored on a scale of one to five, with one representing "not at all" and five representing "very." To familiarize students with the items, they were asked to answer one training question and one practice question (both about happiness) before answering the


Use the pictures below to tell us…

Please circle the words that say how you felt when you did this **math activity:**

(a) (b)

Fig. 2. Tomoji items used in pretests and posttests.
The same set of seven tomoji items, as depicted in the figure, was used in both the pretest and posttest to measure students' math-related emotions. They were presented in the same order to the students, with the three items in (a) appearing first on one page, and the four items in (b) appearing on the next page. The three items in (a) measured happiness, boredom, and challenge, while the four items in (b) measured excitement, nervousness, frustration, and hope.

seven items about math-related emotions.

Measure of rational number skills

Students' math knowledge was assessed through a timed and untimed rational number battery consisting of 44 items used in a previous iteration of Fraction Ball (Bustamante et al., 2022). In the three-minute timed portion of the test, three subsets of rational number skills: *timed fraction to decimal conversion* (eight items), *timed decimal to fraction conversion* (ten items), and *timed fraction addition* (ten items) were assessed, comprising a total of 28 timed items. The time limit was set to assess students' fluency with rational numbers based on prior pilot data of 4th–6th grade students' test completion times. In the untimed part, we measured five subsets: *fraction number line 0 to 1* (four items) and *fraction number line 0 to 5* (four items), where students estimated the position of rational numbers on a 0 to 1 or 0 to 5 number line, respectively; *untimed fraction to decimal conversion* (three items); *untimed decimal to fraction conversion* (three items); and *untimed fraction and decimal addition* (two items), with a total of 16 untimed items. The consistency rates for the timed and untimed items were high at both pretest ($\alpha_{\text{timed}} =$ 0.90 and $\alpha_{\text{untimed}} =$ 0.94) and posttest ($\alpha_{\text{timed}} =$ 0.83 and $\alpha_{\text{untimed}} =$ 0.90). See the average raw scores of each subtest in Appendix Table A1.

We standardized the raw scores using the average grade standard deviation from Grade 4 and Grade 5, with the full sample for pretest scores and only the control group for posttest scores. Then, we calculated the average pretest and posttest standardized composites by averaging the standardized scores across all the subtests. We show the pretest and posttest composite scores in Appendix Table A1. In the current study, only the pretest and posttest overall composites were used (more detailed information about the rational number learning measure and scoring, and the effects of the intervention on rational number learning is provided in Begolli et al., 2023).

Creating positive and negative emotion composites

Average raw scores were calculated for each tomoji item on the pretest and posttest (see Table 1 for descriptive results). Pretest raw scores were standardized using a pooled grade standard deviation (average of fourth- and fifth-grade standard deviation) from the entire sample. Posttest raw scores were also standardized using the pooled standard deviation, but from the control group only. The positive emotion composite is calculated by averaging the standardized scores across the Happy, Excited, and Hopeful items. The negative emotion composite is calculated by averaging the standardized scores across the Bored, Nervous, and Frustrated items. We did not include Challenged in the composite because previous research hasn't suggested a clear-cut valence for it (Karamarkovich & Rutherford, 2021). The internal consistency rates among the three positive emotions ($\alpha_{\text{pretest}} =$ 0.81 and $\alpha_{\text{posttest}} =$ 0.84) were generally higher than the rates for the three negative emotions ($\alpha_{\text{pretest}} =$ 0.63 and $\alpha_{\text{posttest}} =$ 0.62) at pretest and posttest. See Table 1 for the descriptive statistics of the raw scores and Appendix Table A2 for the composite scores.

Data analysis plan

First, to examine the equivalence between the treatment and control groups on measured variables, we regressed students' grade level, gender, attrition, missingness (missing pretest or posttest), and tomoji scores on treatment condition, clustering standard errors by teacher. Next, we used the following statistical models to estimate the impact of the Fraction Ball intervention on students' math-related emotions. Our preferred specifications are regression models treating tomoji results as continuous variables on a scale from one to five, with standard errors clustered at the teacher level, and using grade level and the same tomoji item measured at pretest as covariates. Our outcome of interest is students' self-reported emotion levels, which we refer to as PostTomoji. We pre-registered questions regarding the impact on happiness, nervousness, and positive and negative emotion composites. Nevertheless, we also report impacts on the other five tomoji items to present the breadth of treatment impacts. The outcome variable (PostTomoji) is regressed on treatment assignment to Fraction Ball (FractionBall) while controlling for the same emotion construct measured at pretest (PreTomoji). In other words, for each of the different emotion and composite outcomes at posttest, the pretest score corresponding to the same outcome was entered as a covariate. Therefore, our first regression equation is as follows:

Table 1

Descriptive statistics of fraction ball moderators and tomoji raw scores.

	Full Sample			Control			Treatment				
Variable	*N*	Mean	*SD*	*N*	Mean	*SD*	*N*	Mean	*SD*	*b*	*p*
4th Grade	360	0.61		162	0.59		198	0.62		0.03	0.89
5th Grade	360	0.39		162	0.41		198	0.38		−0.03	0.89
Female	333	0.52		148	0.45		185	0.58		0.13	0.00*
Attrition from pretest to posttest	342	0.08		151	0.10		191	0.06		−0.04	0.17
Missing pretest	360	0.05		162	0.07		198	0.04		−0.03	0.25
Missing posttest	360	0.07		162	0.09		198	0.06		−0.04	0.19
Pretest											
Tomoji Happy	335	3.01	1.20	146	3.08	1.17	189	2.95	1.23	−0.14	0.43
Tomoji Excited	336	2.88	1.31	149	2.96	1.27	187	2.81	1.34	−0.15	0.45
Tomoji Hopeful	337	3.27	1.25	150	3.33	1.21	187	3.22	1.28	−0.11	0.42
Tomoji Bored	337	2.60	1.24	149	2.47	1.17	188	2.71	1.28	0.24	0.29
Tomoji Nervous	336	2.79	1.42	150	2.69	1.39	186	2.87	1.45	0.18	0.37
Tomoji Frustrated	336	2.39	1.37	151	2.10	1.17	185	2.63	1.47	0.53	0.01*
Tomoji Challenged	337	3.19	1.15	148	3.09	1.09	189	3.28	1.19	0.19	0.37
Posttest											
Tomoji Happy	332	2.95	1.20	146	2.82	1.12	186	3.06	1.25	0.24	0.27
Tomoji Excited	332	2.68	1.25	147	2.61	1.25	185	2.73	1.26	0.12	0.48
Tomoji Hopeful	333	3.25	1.23	147	3.07	1.21	186	3.38	1.24	0.31	0.12
Tomoji Bored	329	2.69	1.24	144	2.83	1.16	185	2.58	1.28	−0.25	0.27
Tomoji Nervous	332	2.42	1.37	147	2.39	1.41	185	2.45	1.33	0.05	0.83
Tomoji Frustrated	333	2.15	1.29	147	2.16	1.30	185	2.14	1.28	−0.02	0.93
Tomoji Challenged	328	2.90	1.20	142	2.85	0.91	186	2.94	1.17	0.09	0.63

P-value is based on regressing each variable on treatment condition, clustering standard errors by teacher. The *N* in the Variable section refers to the total sample possible, including student attrition. * $p < .05$, ** $p < .01$.

$$\text{PostTomoji (e.g., Posttest Happiness)} = \beta 0 + \beta 1 \text{ FractionBall} + \beta 2 \text{ PreTomoji (e.g., Pretest Happiness)} + \beta 3 \text{ Grade4} + \varepsilon \text{ (standard errors were clustered at the teacher level to account for the nonindependence of students within classrooms)}$$

The estimand β1 represents the estimated causal effect of the Fraction Ball intervention on the treatment group students' math-related emotion outcomes, holding their grade and pretest emotion levels constant.

In addition, we report estimates that are not adjusted for covariates (e.g., pretest tomoji and grade level) to test the sensitivity of estimates to the inclusion of baseline covariates. Furthermore, as the pretest frustration level was already statistically significantly different before at pretest (the treatment group has a higher level, as shown in Table 1), we also report the estimates from models adding the pretest frustration level as an additional control.

$$\text{PostTomoji} = \beta 0 + \beta 1 \text{ FractionBall} + \beta 2 \text{ PreTomoji} + \beta 3 \textit{ PreFrustration} + \beta 4 \text{ Grade4} + \varepsilon \text{ (standard errors clustered at the teacher level)}$$

As robustness checks, we employed generalized linear mixed models (GLMMs) with random intercepts by teachers (Appendix Table A3 and A4). In our preferred models with clustered standard errors at the teacher level, we assume classrooms have equivalent errors. In GLMMs, intercepts are calculated as the average standard error and a measure of variance for each specific classroom. Moreover, these models do not include children with missing responses. To further assess the robustness of our estimates, we employed regression models using full information maximum likelihood (FIML) to handle missing data and estimate treatment effects for the entire sample ($N = 360$; Appendix Table A5).

In addition, we conducted pre-registered interaction analyses using the preferred regression models to investigate whether students' pretest emotion levels moderate the effects of the intervention. Specifically, we included an interaction term between the treatment assignment and pretest emotion levels in the following regression:

$$\begin{aligned}\text{PostTomoji} = {} & \beta 0 + \beta 1 \text{ FractionBall} + \beta 2 \text{ PreTomoji} \\ & + \beta 3 \textit{ FractionBall}^{*} \textit{PreTomoji} + \beta 4 \text{ Grade4} \\ & + \varepsilon \text{ (standard errors clustered at the teacher level)}\end{aligned}$$

As pre-registered, we used the pretest negative emotion composite and nervousness as moderators. We were particularly interested in whether Fraction Ball was effective for students who rated themselves as high on nervousness at baseline. In order to more intuitively present the varying treatment impacts for students with differing levels of negative emotion at pretest, we performed two median splits: one based on pretest nervousness level and the other based on the pretest negative emotion composite score. To explore if students' pretest nervousness moderated the treatment impacts, we first divided our sample into two groups based on whether individual students' pretest negative emotion composites were above or below the median value of the whole sample ($n_{high} = 170$; $n_{low} = 170$). Then, we estimated the treatment impacts using our preferred model separately for the two subgroups (high and low pretest negative emotion subgroups). Then, we repeated the aforementioned process to conduct a median split using students' pretest nervousness scores ($n_{high} = 122$; $n_{low} = 214$) and estimated the subgroup treatment effects. The moderation roles of student pretest negative emotion composites and nervousness were robust to the different modeling decisions (i.e., median splits and continuous moderators; see *Moderation of Treatment Impacts* subsection under Results). In addition, we tested the moderating roles of student grade level and gender following our pre-registration.

Finally, we explored the associations between emotion and rational number learning gains for each block of classrooms. We estimated the effects on the overall composite scores of the rational number learning, positive emotion composite, and negative emotion composite for each block of classrooms using the following equation:

$$\begin{aligned}\text{PostComposite} = {} & \beta 0 + \beta 1 \text{ PreComposite} + \beta 2 \text{ FractionBall} + \beta 3 \text{ Block} \\ & + \beta 4 \textit{ FractionBall}^{*} \textit{Block} + \beta 5 \text{ Grade4} \\ & + \varepsilon \text{ (standard errors clustered at the teacher level)}\end{aligned}$$

Then, we created plots to visualize the relationships between the estimated effects on two different outcomes for the eight blocks and estimated the slopes of the fitted regression lines, which represent the model-predicted rates of change of the impact on one outcome (e.g., rational number learning) with respect to the impact on the other outcome (e.g., positive emotions) at the classroom block level.

Results

Descriptive statistics are presented in Table 1 for tomoji raw scores by group and in Appendix Table A2 for standardized composite scores. Separate regression analyses on tomoji items and standardized composite scores, clustering errors by teacher, showed the intervention and control groups were not statistically different in most tomoji items, attrition, missingness, or grade level ($ps > 0.05$). However, the intervention group had statistically significantly more female students ($b = 0.13$, $p < .01$) and higher levels of frustration than the control group ($b = 0.53$, $p < .01$) at pretest. In other words, as shown in Table 1, the baseline equivalence was established for most demographic variables and tomoji items with the exception of gender and pretest frustration.

Average math-related emotion levels at posttest (RQ1)

We pre-registered our hypotheses for RQ1: treatment group students' average self-reported feelings of happiness at posttest would range from three to five, and their nervousness levels would range from one to two. As shown in Table 1, the treatment group's average happiness level at posttest was 3.06, which was slightly higher than the control group's rating (2.82), consistent with what we hypothesized. This result shows that students in the treatment group, on average, rated themselves somewhere between "kind of happy" and "happy." The average nervousness level was 2.45, slightly higher than the control group's rating (2.39) and also higher than hypothesized. This finding suggests that treatment group students rated them somewhere between "a tiny bit nervous" and "kind of nervous."

Fraction ball treatment impacts on math-related emotions (RQ2)

The treatment impact estimates of Fraction Ball on tomoji outcomes are presented in Table 2. The first model does not include any covariates, and we report the estimates to explore if they are sensitive to the inclusion of the covariates. The second model with pretest tomoji score and grade level as covariates is our pre-registered preferred specification. The third model includes an additional control - pretest frustration level - as it differed statistically significantly between treatment and control groups at pretest.

As shown in the first model in Table 2, when no covariates were included in the model, the estimated effects of the intervention on posttest tomoji were not statistically significant ($ps > 0.05$). Once students' tomoji levels on the pretest and grade level were added as covariates, standard errors became slightly smaller, and the estimated effects became larger and statistically significant on three outcomes, including the positive emotion composite ($b = 0.28$, $p = .03$), the *hopeful*

Table 2
Estimated treatment effects.

	No Covariates				Pretest Tomoji & Grade Covariates				Pretest frustration, Pretest Tomoji & Grade Covariates			
Standardized Posttest Outcome	*N*	*b*	*(SE)*	*p*	*N*	*b*	*(SE)*	*p*	*N*	*b*	*(SE)*	*p*
Pre-registered outcomes												
Happy	332	0.21	(0.18)	0.27	309	0.30	(0.15)	0.07	305	0.29	(0.16)	0.08
Nervous	332	0.04	(0.19)	0.83	308	−0.02	(0.13)	0.88	307	−0.07	(0.14)	0.64
Positive emotion composite	333	0.19	(0.14)	0.20	313	0.28	(0.11)	0.03*	308	0.28	(0.12)	0.03*
Negative emotion composite	334	−0.06	(0.16)	0.72	314	−0.20	(0.11)	0.07	310	−0.20	(0.11)	0.09
Not Pre-registered outcomes												
Excited	332	0.10	(0.13)	0.48	308	0.20	(0.10)	0.06	305	0.21	(0.11)	0.08
Hopeful	333	0.25	(0.15)	0.12	310	0.30	(0.13)	0.04*	310	0.32	(0.06)	0.03*
Bored	329	−0.21	(0.19)	0.27	308	−0.33	(0.14)	0.04*	304	−0.35	(0.14)	0.02*
Frustrated	333	−0.02	(0.19)	0.92	309	−0.18	(0.13)	0.19	309	−0.18	(0.13)	0.19
Challenged	328	0.07	(0.15)	0.63	308	0.03	(0.13)	0.79	304	0.00	(0.13)	0.99

Standardized scores are reported to allow for comparison across measures. The pretest standardized scores were calculated using the average grade standard deviation from Grade 4 and Grade 5. The posttest standardized scores were calculated using the average grade standard deviation but from the control group only. Positive emotion composite is the average standardized scores of Happy, Excited, and Hopeful items. Negative emotion composite is the average standardized scores of Bored, Nervous, and Frustrated items. Clustered standard errors by teachers are in parentheses. The second model is our pre-registered preferred model with pretest tomoji score and grade level covariates. * $p < .05$.

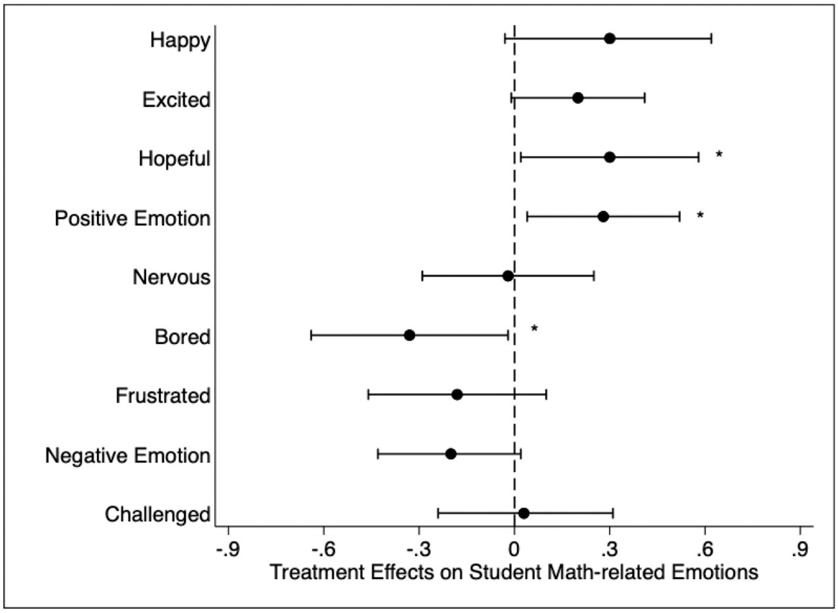


Fig. 3. Estimated treatment outcomes on different math-related emotions.
Estimates represent regression coefficients for treatment status. The estimates can be interpreted as standardized treatment effects in pooled grade SDs of the control group. Models are specified using clustered standard errors by teacher, controlling for pretest tomoji score and grade level. Bars show 95% confidence intervals. * $p < .05$.

item ($b = 0.30$, $p = .03$), and the *bored* item ($b = -0.30$, $p = .02$). The results showed that the impact estimate of participating in the Fraction Ball intervention was 0.28 SD on the positive emotion composite, 0.30 SD for the level of feeling hopeful, and 0.30 SD for the level of feeling bored. The treatment estimates on all the standardized outcomes from the preferred model are also shown in Fig. 3. Moreover, after accounting for the pretest frustration level (see the third model in Table 2), the estimates were very similar in magnitude to our preferred model. They also followed the same pattern of statistical significance or non-significance.

Robustness check

First, we estimated models with random intercepts by teacher to check for the robustness of our results. Appendix Table A3 shows estimated treatment effects from the random intercept models, which were similar in magnitude to the estimates from the regression models with covariates and clustering standard errors by teachers. Nevertheless, the pattern of statistical significance/non-significance changed for two outcomes: the Fraction Ball intervention impact on the feeling of happiness ($b = 0.31$, $p = .047$) and excitement ($b = 0.20$, $p = .045$) became statistically significant. Appendix Table A4 shows estimated fixed and random effects components for the random intercept model with the positive and negative emotion composite scores as the outcome. The results were based on two models: the baseline model without predictor and our preferred model with pretest and grade covariates. Results from the baseline model indicated statistically significant variability in students' positive and negative emotion composite scores at the teacher level and student level within classrooms. Furthermore, 11% of the residual variance in students' negative emotion composite scores was due to differences between teachers/classrooms and 6% for the positive emotion composite. After adding the respective pretest composite score and grade level as covariates into the model, the residual

Table 3

Estimated treatment by moderator effects of pretest nervousness and negative emotion composite.

Models	*N*	*b*	*(SE)*	*p*
Prior Nervousness				
Pretest nervousness	308	0.59	(0.04)	0.00***
Treatment	308	−0.02	(0.13)	0.88
Pretest nervousness X Treatment	308	−0.17	(0.07)	0.04*
4th grade	308	0.21	(0.13)	0.40
Prior Negative Emotion Composite				
Pretest negative emotion composite	314	0.75	(0.07)	0.00***
Treatment	314	−0.21	(0.10)	0.06
Pretest negative emotion composite X Treatment	314	−0.21	(0.09)	0.02*
4th grade	314	0.05	(0.10)	0.64

The results are based on two separate models with posttest nervousness and negative emotion composite as outcomes. The outcomes are regressed on respective interaction terms and main effects (treatment X pretest nervousness and treatment X pretest negative emotion composite) and grade level. Standardized scores are reported to allow for comparison across measures. The pretest standardized scores were calculated using the average grade standard deviation from Grade 4 and Grade 5. The posttest standardized scores were calculated using the average grade standard deviation but from the control group only. The standardized negative composite score is the average standardized scores of Bored, Nervous, and Frustrated. Clustered standard errors by teacher are in parentheses. * $p < .05$, *** $p < .001$.

variance due to teacher differences was slightly reduced to 7% for the negative emotion composite and 5% for the positive emotion composite. The variability between teachers/classrooms and students within classrooms remained statistically significant but was slightly smaller in magnitude.

As demonstrated in Appendix Table A5, the models using FIML to handle missing data and estimate the effects using the full sample ($N = 360$) showed similar results to our preferred models. Nevertheless, although the estimated effect on the negative emotion composite stayed very similar in magnitude, the estimated impact became statistically significant ($b = -0.21$, $p = .04$).

Moderation of treatment impacts (RQ3)

We employed our preferred regression models and included interaction terms between the treatment condition and students' characteristics (i.e., pretest emotion level, grade level, and gender) to investigate whether these factors moderated the intervention impacts on math-related emotions. For students' prior emotions, we pre-registered to examine the moderating role of pretest nervousness level and negative emotion composite scores, and the results of the two models are shown in Table 3. We found that both the pretest nervousness (*treatmentXpretest* $b = -0.17$, $p = .04$) and negative emotion levels (*treatmentXpretest* $b = -0.21$, $p = .02$) moderated the effect of the intervention on the respective emotion level at posttest. The intervention impacts were larger for students with higher pretest nervousness (for students above median pretest nervousness, $b = -0.13$, $p > .05$) or negative emotion levels (for students above median pretest negative emotions, $b = -0.32$, $p = .02$) compared to students with lower pretest nervousness (for students at or below median pretest nervousness, $b = 0.07$, $p > .05$) or negative emotion levels (for students at or below median pretest negative emotions, $b = -0.10$, $p > .05$). In addition, we examined the moderating effects of student grade level and gender. As presented in Appendix Table A6, we did not find that the grade levels and gender moderated the intervention impacts ($ps > 0.05$).

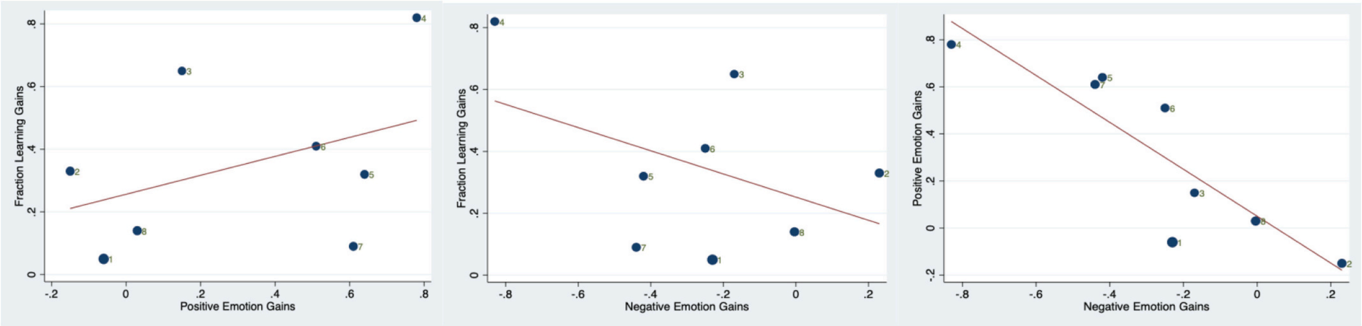
Associations between emotion and learning outcomes (RQ4)

We conducted an exploratory analysis to examine the relationships between the estimated impact of the Fraction Ball intervention and different outcome domains, including math learning and social-emotional outcomes. Fig. 4 shows three scatter plots that illustrate the estimated results of the eight classroom blocks. Plot (a) indicates a positive association between impacts on positive emotion and rational number learning. The estimated slope of the fitted regression line is 0.34, which indicates that a 0.1 SD higher impact on increasing positive emotion is associated with a 0.034 SD higher impact on increasing rational number skills at the classroom block level. Plot (b) shows a negative association between impacts on negative emotion and rational number learning. The estimated slope of the regression line is −0.48, which indicates that 0.1 SD higher impact on decreasing negative emotion is associated with a 0.048 SD higher impact on increasing rational number skills at the classroom block level. Finally, Plot (c) demonstrates a negative relationship between the impacts on positive emotion and negative emotion. The fitted regression line's slope is −1.16, which suggests that a 0.10 SD higher impact on decreasing negative emotion is statistically significant associated with 0.116 SD higher impact on increasing positive emotion at the classroom block level. Overall, the results suggest that Fraction Ball did not yield trade-offs between impacts on student rational number learning and emotional outcomes at the classroom block level.

Discussion

This study presented an innovative and effective approach to addressing negative and promoting positive emotional experiences associated with mathematics among elementary school students. Specifically, we implemented a physically active and socially engaging math intervention—Fraction Ball—with the aim of fostering positive math-related emotions while facilitating rational number learning. Our findings demonstrate that Fraction Ball did positively affect emotions. We observed that the Fraction Ball intervention yielded effects on 4th and 5th graders' positive emotion composite corresponding to approximately ¼ of one point on our five-point scale, in addition to specifically increasing hopefulness and decreasing boredom by approximately $^1/_3$ of one point on our five-point scale. These findings highlight Fraction Ball's potential for improving students' emotional states as well as math learning outcomes when considering the positive impacts of the intervention on rational number skills in our previous studies (Begolli et al., 2023; Bustamante et al., 2022).

In addition, our results are consistent with developmental theories pertaining to affordances of play and play-based learning. Theories of playful learning have posited that making learning contexts fun and engaging will encourage and support high-quality learning (Hirsh-Pasek et al., 2015), which is supported here by the positive impacts on both rational number learning and math-related emotions, the latter of which indicates the playful experiences were indeed fun. We provide evidence that when playful learning activities are thoughtfully designed, students' learning and emotional outcomes are not at odds with each other, even when engaging with challenging topics (e.g., rational numbers) and tasks (e.g., speeded practice). This extends the playful learning field by providing a test of whether play can support learning and positive emotion development in mathematics, even with this set of learning goals in a domain that is often associated with negative emotions. Our



(a) (b) (c)

Fig. 4. Associations between the estimated treatment effects on learning and emotional outcomes for each classroom block.
Three scatter plots show the relationships between the estimated impacts on different outcomes of the Fraction Ball intervention for each of the eight classroom blocks. Plot (a) shows an overall positive association between estimated impacts on positive emotion composite and rational number learning. Plot (b) shows an overall negative association between estimated impacts on negative emotion composite and rational number learning. Plot (c) shows an overall negative association between estimated impacts on negative emotion composite and positive emotion composite. The red lines represent fitted regression lines modeling the linear relationships between impacts on two different outcomes. Each blue marker point represents a block of classrooms. The number of the block is labeled next to the marker point. The size of the marker point is proportional to the size of the two classrooms within the block.

use of emotion measures at the same time as specific mathematical outcomes provides data-driven insights into this positive relationship between play, positive emotions, and mathematical learning.

Further, by examining the impacts on different types of emotions, this paper aligns with the call to move beyond math anxiety when examining the relations between mathematics learning and affect and emphasizes positive emotions in math learning (Villavicencio & Bernardo, 2016). Specifically, for positive emotions, we focused on happy, excited, and hopeful, all of which are grounded in the control-value theory of achievement emotions (Pekrun, 2006) and have been studied in math learning contexts (e.g., Karamarkovich & Rutherford, 2021). Our findings indicate that the Fraction Ball intervention effectively increased the level of hope among students in the treatment group compared to those in the control group. The intervention's impact on students' improved hope might be attributed to the theoretical connection between hope and the sense of control over achievement goals (Pekrun, 2006). According to the Control-Value theory of academic emotions, hope is categorized by individuals' partial control of an academic task and the perception of an attainable positive outcome (Pekrun et al., 2007). Therefore, one possible underlying mechanism for this improvement in hope is the students' increased sense of control and perception of the likelihood of achieving positive outcomes in math learning. This change can result from their successful experiences in solving fraction and decimal problems, both during the Fraction Ball activities and the subsequent assessments facilitated by the intervention.

We also found statistically significant effects on the overall positive emotion composite scores (averages of the levels of happiness, excitement, and hope). This finding aligns with our proposed mechanism, rooted in the theoretical links between play and positive emotions (e.g., Kuczaj & Horback, 2012), indicating a play-based intervention has the potential to create positive emotional experiences. Moreover, our study contributes empirical evidence that integrating playful learning into math education can not only produce improvements in learning but also positive emotional outcomes.

Nevertheless, we did not observe statistically positive impacts on happiness and excitement individually. These two emotions are closely associated with enjoyment (Di Leo, Muis, Singh, & Psaradellis, 2019). One potential explanation for the lack of effects on these specific emotions could be attributed to the context in which students filled out the emotion items. Specifically, they completed the emotion assessment immediately after an approximately 25-min rational number knowledge test. As a result, it is plausible that students may have felt hopeful that they performed well on this test but not activating emotions such as enjoyment.

In terms of negative emotions, we assessed boredom, frustration, and nervousness, and found statistically significant reductions in feelings of boredom among students in the treatment group compared to the control group. Previous research suggests that boredom is a relatively common feeling experienced by students in math classes (e.g., Daschmann, Goetz, & Stupnisky, 2011). The significant decrease in boredom levels we observed might be explained by the main factors of boredom as discussed in Hanin and Van Nieuwenhoven (2016). The authors suggest that students tend to feel bored when faced with "routine, non-challenging tasks" (p. 135) that fall below their zone of proximal development. In contrast, the Fraction Ball activities, as mentioned earlier, centered on play and games that have been theorized to increase students' motivation and attention (Hassinger-Das et al., 2017), which in turn may make it less likely for students to feel bored during the activities. Moreover, our Fraction Ball games were designed to offer students opportunities for autonomy, along with scaffolding from teachers in a playful environment. Students might feel more engaged during the games and thus report reduced boredom during math activities after the intervention. Thus, our findings may offer new theoretical insights into the power of playful learning in enhancing both learning outcomes and reducing negative emotions, specifically decreasing boredom, in the context of math education.

Further, our findings suggested that the intervention did not result in observable effects on students' self-reported level of feeling challenged. The existing research on the emotion of feeling challenged has yielded mixed opinions, such as not clear-cut valence (positive or negative), which may be interpreted differently for different students (Karamarkovich & Rutherford, 2021). Therefore, the null effects we observed might also stem from the heterogeneity in how students interpreted the challenged item. On one hand, feeling challenged was positively associated with hope and negatively related to boredom (Kirby, Morrow, & Yih, 2014; Pekrun et al., 2007). Therefore, students who view challenge as a positive emotion might report increased levels of it, potentially reflecting an increased willingness to exert more effort to achieve success in math because of Fraction Ball. Conversely, feeling challenged has also been found to be associated with negative feelings (Kirby et al., 2014). Thus, as Fraction Ball mitigated students' negative math-related emotions, individuals interpreting challenge as a negative emotion might report reduced levels of it. However, all of this is speculative. Future research on feeling challenged can focus on exploring the underlying factors and examining its interactions with related emotions to gain a deeper understanding of their complex nature and could provide insights relevant to intervention design.

In addition, we found that the Fraction Ball intervention was more potent for students who exhibited higher levels of negative emotions on the pretest. In other words, students with initially elevated negative emotion levels experienced larger experimental impacts on decreasing negative emotions when they received the Fraction Ball treatment. This finding is exciting because it suggests that this intervention in specific, and perhaps play-based interventions in general, may particularly benefit students who struggle with generalized negative emotions about mathematics. Thus, playful math activities that provide a low-stress learning environment that normalizes errors and encourages active engagement might contribute to a more equitable math learning experience. This finding is consistent with previous work indicating that learning from common errors in algebra (Booth et al., 2015) and basketball practices (Nasir & Hand, 2008) could be especially beneficial for underrepresented minority students. Teacher feedback as well as peer feedback loops in the roles of the tracker and checker discussed in the introduction may have served to challenge misconceptions, provide correct concepts, and afford deep engagement for students in a flexible, playful environment. Scholars have emphasized the need to develope math activities that can not only reinforce the interests of students with a preexisting high motivation but also encourage students with negative emotions about mathematics to engage in math classes (Beilock & Maloney, 2015).

Moreover, our exploratory analysis showed that classrooms that demonstrated greater rational number learning gains also showed larger math-related emotion gains, suggesting that, at least for classrooms participating in Fraction Ball, improvements in these outcomes were not at odds with each other, despite the speeded practice students received during the intervention. These findings, overall, provide an impetus for future studies expanding the use of Fraction Ball and other play-based learning experiences in mathematics instruction, with likely implications for both affective and mathematics learning outcomes. Future work will be conducted to unpack the potential pathways suggested by our findings. These pathways include: 1) playful learning increases math skills, which in turn reduces negative math-related emotions and increases positive math-related emotions, 2) playful activities first improve emotional experiences and thereby allow for an increase in math learning, and 3) playful learning positively improves both outcomes through largely independent mechanisms. Regardless of what combination of these mechanisms produced such impacts, our findings represent a clear example of a case in which an intervention that used speeded practice and included timed assessments improved both students' learning and math-related emotions. Future discussions of the appropriate role of these practices should consider the potential roles of (thoughtful) design and implementation in managing potential trade-offs between optimizing for cognitive and social-emotional outcomes.

Limitations

Our study contributes to the literature on playful learning in math education by demonstrating evidence that a play-based intervention focused on rational number intervention could improve students' math-related emotions, in addition to enhancing their rational number skills as found in Begolli et al. (2023). However, some limitations are important to consider when interpreting the results. First, due to the experimental nature of the study, it falls beyond our scope to identify the exact level(s) of specificity of the emotion measure (e.g., math-related emotions vs. achievement emotions). It could be possible that the intervention elicited a broader impact, potentially improving students' achievement emotions. Conversely, the impact could be narrowly focused on students' emotions specifically related to rational numbers. Second, similarly, while Fraction Ball activities might enhance certain social-emotional skills (e.g., emotion regulation and relationship skills) among students through interactive engagement, we did not gather data on a wide range of social-emotional skills to investigate these potential impacts. Further work on playful learning activities might benefit from collecting such measures to build a better mechanistic understanding of the pathways through which playful learning might influence children's social-emotional outcomes. Lastly, in the current version of the tomoji measure, we included only two training items focused on positive emotions. Although we are not sure how this may have affected students' responses, it is plausible that erroneous responses may be mitigated by including a negative practice item as well. We plan to pilot a version with a training item using negative emotions in the future.

Constraints on generality

Our findings indicate positive effects of engaging in playful rational number activities on students' math-related emotions. Although the Fraction Ball activities were co-designed with local school teachers, for the current experiment, we excluded co-designed schools. As a result, the participating schools, teachers, and students had no prior exposure to the Fraction Ball game or co-design activities. Therefore, we expect the key elements characterizing the Fraction Ball program, in particular its playful and active learning components, will hold promise for enhancing student emotional outcomes in similar school districts that primarily serve Latine and low-income students. However, it is beyond the scope of this work to have evidence to support the generalization of these findings to more diverse school settings with differences in school district structure or student population composition. We have no reason to believe that the results depend on other characteristics of the student or teacher participants, intervention materials, or context. Finally, further work is needed to explore the generality of our observed impact to other math activities and interventions.

Conclusion

The results of our experiment provide promising evidence that the Fraction Ball intervention can be an effective strategy to enhance students' math-related emotions among elementary school students, particularly those who initially demonstrated more negative math-related emotions. This is a potentially powerful finding, lending support to the field of play-based learning theory and indicating that play combined with more traditional math instruction may increase both learning and positive emotions without trade-offs that would have meant that negative emotions were required for learning. Future research can delve into the possible mechanisms of these heterogeneous treatment impacts of the Fraction Ball intervention and thus provide implications for future math activity development. Understanding how to address a broader range of math-related emotions will be useful for developing effective interventions that address negative math-related emotions in complex math topics such as rational number concepts.

CRediT authorship contribution statement

Siling Guo: Writing – original draft, Visualization, Methodology, Formal analysis, Conceptualization. **Drew H. Bailey:** Writing – review & editing, Supervision, Methodology. **Katherine Rhodes:** Writing – review & editing, Conceptualization. **Kreshnik Nasi Begolli:** Writing – review & editing, Project administration, Conceptualization. **Vanessa N. Bermudez:** Writing – review & editing, Project administration, Investigation. **LuEttaMae Lawrence:** Writing – review & editing, Project administration, Investigation. **Daniela Alvarez-Vargas:** Writing – review & editing, Project administration, Investigation. **Lourdes M. Acevedo-Farag:** Writing – review & editing, Conceptualization. **June Ahn:** Writing – review & editing, Conceptualization. **Andres S. Bustamante:** Writing – review & editing, Project administration, Funding acquisition, Conceptualization. **Lindsey E. Richland:** Writing – review & editing, Supervision, Conceptualization.

Declaration of competing interest

The authors report there are no competing interests to declare.

Data availability

The authors do not have permission to share data.

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Appendix A. Appendix

Table A1 Descriptive statistics of rational number skill subtest raw scores and composite scores (Adapted from Begolli et al., 2023).

	Full Sample			Control			Treatment				
Variable	*N*	Mean	*SD*	*N*	Mean	*SD*	*N*	Mean	*SD*	*b*	*p*
Pretest											
Timed fraction to decimal conversion	342	10%	22%	151	14%	27%	191	7%	17%	−7.36	0.39
Timed decimal to fraction conversion	342	20%	24%	151	20%	22%	191	19%	25%	−1.01	0.90
Timed fraction addition	342	48%	29%	151	48%	29%	191	47%	29%	−0.50	0.94
Untimed fraction to decimal conversion	342	9%	21%	151	12%	25%	191	6%	15%	−6.78	0.32
Untimed decimal to fraction conversion	342	39%	45%	151	36%	43%	191	41%	47%	4.41	0.77
Untimed fraction and decimal addition	342	25%	37%	151	33%	42%	191	18%	32%	−14.79	0.25
PAE fraction number line 0 to 1	342	21%	18%	151	22%	18%	188	21%	17%	−1.81	0.67
PAE fraction number line 0 to 5	342	28%	16%	151	26%	15%	188	29%	16%	3.27	0.43
Overall composite	342	−0.00	0.71	151	0.08	0.78	191	−0.06	0.64	−0.14	0.60
Posttest											
Timed fraction to decimal conversion	334	21%	32%	147	19%	33%	187	22%	32%	2.52	0.84
Timed decimal to fraction conversion	334	28%	30%	147	27%	32%	187	29%	28%	2.12	0.84
Timed fraction addition	334	56%	27%	147	56%	29%	187	55%	26%	−0.91	0.89
Untimed fraction to decimal conversion	334	33%	40%	147	23%	37%	187	41%	41%	18.68	0.16
Untimed decimal to fraction conversion	334	48%	46%	147	44%	47%	187	50%	45%	5.87	0.64
Untimed fraction and decimal addition	334	39%	41%	147	34%	41%	187	44%	41%	9.91	0.45
PAE fraction number line 0 to 1	324	17%	17%	141	20%	17%	183	15%	16%	−4.86	0.22
PAE fraction number line 0 to 5	324	23%	15%	141	24%	16%	183	22%	14%	−1.79	0.69
Overall composite	334	0.00	0.85	147	−0.11	0.87	187	0.08	0.82	0.19	0.54

The subtests raw scores are presented in percentages, indicating the proportion of students who answered correctly (except for PAEs). PAEs represent percentages of absolute errors. The overall composite scores are the average standardized scores across all subtests. *P*-value is based on regressing each variable on treatment condition, clustering standard errors by teacher.

Table A2 Summary statistics of composite scores of Tomoji outcomes.

	Full Sample			Control			Treatment				
Construct	*N*	Mean	*SD*	*N*	Mean	*SD*	*N*	Mean	*SD*	*b*	*p*
Pretests											
Positive emotion composite	340	0.01	0.85	150	0.06	0.82	190	−0.04	0.88	−0.10	0.41
Negative emotion composite	340	0.00	0.76	151	−0.13	0.68	189	0.11	0.80	0.25	0.06
Posttests											
Positive emotion composite	333	0.00	0.90	147	−0.10	0.86	186	0.09	0.92	0.19	0.20
Negative emotion composite	334	0.00	0.78	147	0.03	0.77	187	−0.02	0.79	−0.06	0.67

P-value is based on regressing each variable on treatment condition, clustering standard errors by teacher. The pretest standardized scores were calculated using the average grade standard deviation from Grade 4 and Grade 5. The posttest standardized scores were calculated using the average grade standard deviation but from the control group only. Positive emotion composite is the average standardized scores of Happy, Excited, and Hopeful items. Negative emotion composite is the average standardized scores of Bored, Nervous, and Frustrated items.

Table A3 Estimated treatment effects from general linear mixed models using random intercepts by teacher ($N_{teachers} = 16$).

	No Covariates				Pretest Tomoji & Grade Covariates				Pretest Frustration, Pretest Tomoji & Grade Covariates			
Standardized Posttest Outcome	*N*	*b*	*(SE)*	*p*	*N*	*b*	*(SE)*	*p*	*N*	*b*	*(SE)*	*p*
Pre-registered Outcomes												
Happy	332	0.23	(0.18)	0.20	309	0.31	(0.16)	0.05*	305	0.31	(0.16)	0.05*
Nervous	332	0.01	(0.18)	0.96	308	−0.03	(0.13)	0.83	307	−0.07	(0.13)	0.56
Positive emotion composite	333	0.22	(0.16)	0.16	313	0.28	(0.12)	0.02*	308	0.29	(0.12)	0.02*

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(*continued*)

	No Covariates				Pretest Tomoji & Grade Covariates				Pretest Frustration, Pretest Tomoji & Grade Covariates			
Standardized Posttest Outcome	*N*	*b*	*(SE)*	*p*	*N*	*b*	*(SE)*	*p*	*N*	*b*	*(SE)*	*p*
Negative emotion composite	334	−0.10	(0.19)	0.61	314	−0.21	(0.11)	0.06	310	−0.19	(0.11)	0.08
Not Pre-registered Outcomes												
Excited	332	0.10	(0.13)	0.45	308	0.20	(0.10)	0.05*	305	0.21	(0.11)	0.05
Hopeful	333	0.26	(0.15)	0.07	310	0.30	(0.13)	0.02*	304	0.32	(0.14)	0.02*
Bored	329	−0.20	(0.17)	0.24	308	−0.32	(0.14)	0.02*	304	−0.34	(0.15)	0.02*
Frustrated	333	−0.05	(0.18)	0.79	309	−0.19	(0.14)	0.16	309	−0.19	(0.14)	0.16
Challenged	328	0.07	(0.13)	0.62	308	0.03	(0.11)	0.76	304	0.00	(0.11)	0.98

Models are specified using random intercepts at the teacher level. Standardized scores are reported to allow for comparison across measures. Pretest raw scores were standardized using the average grade standard deviation from Grade 4 and Grade 5. Posttest raw scores were standardized using the average grade standard deviation but from the control group. The standardized positive composite score is the average of Happy, Excited, and Hopeful standardized scores. The standardized negative composite score is the average of Bored, Nervous, and Frustrated standardized scores. Clustered standard errors by teacher are in parentheses. * $p < .05$.

Table A4 General linear mixed model with negative and positive emotion composite as the outcome ($N_{teachers} = 16$).

Model	No Predictor				Pretest Tomoji & Grade Covariates			
	N	*b*	*(SE)*	*p*	*N*	*b*	*(SE)*	*p*
Negative Emotion Composite Model								
Fixed effects								
Intercept	334	0.00	(0.08)	0.16	314	0.07	(0.10)	0.45
Treatment						−0.20	(0.11)	0.06
Pretest negative emotion composite						0.59	(0.05)	0.00***
Grade4						0.06	(0.10)	0.45
Error Standard Deviation								
Teacher Intercept		0.26	(0.07)	0.00***		0.16	(0.05)	0.00***
Residual		0.73	(0.03)	0.00***		0.60	(0.02)	0.00***
Intraclass correlation		0.11	(0.05)			0.07	(0.04)	
Positive Emotion Composite Model								
Fixed effects								
Intercept	333	0.00	(0.07)	0.96	313	−0.17	(0.11)	0.11
Treatment						0.28	(0.12)	0.02*
Pretest positive emotion composite						0.59	(0.05)	0.00***
Grade4						0.05	(0.11)	0.65
Error Standard Deviation								
Teacher Intercept		0.23	(0.07)	0.00***		0.17	(0.06)	0.00***
Residual		0.87	(0.03)	0.00***		0.70	(0.03)	0.00***
Intraclass correlation		0.06	(0.04)			0.05	(0.04)	

The results are based on two separate models with posttest positive and negative emotion composite as outcomes. The first model doesn't have any predictor variable, and the second model includes treatment status, respective pretest composite, and grade level as covariates. Models are specified using random intercepts at the teacher level. Standardized scores are reported to allow for comparison across models. Adjusted standard errors are in parentheses. The pretest standardized scores were calculated using the average grade standard deviation from Grade 4 and Grade 5. The posttest standardized scores were calculated using the average grade standard deviation but from the control group only. Positive emotion composite is the average standardized scores of Happy, Excited, and Hopeful items. Negative emotion composite is the average standardized scores of Bored, Nervous, and Frustrated items. * $p < .05$, *** $p < .001$.

Table A5 Estimated treatment effects using full information maximum likelihood for missing data.

	Pretest Tomoji & Grade Covariates			
Standardized Posttest Outcome	*b*	*(SE)*	*p*	*N*
Pre-registered Outcomes				
Happy	0.28	(0.15)	0.06	360
Nervous	−0.02	(0.13)	0.86	360
Positive emotion composite	0.27	(0.11)	0.01*	360
Negative emotion composite	−0.21	(0.11)	0.04*	360
Not Pre-registered Outcomes				
Excited	0.17	(0.10)	0.09	360
Hopeful	0.31	(0.12)	0.01*	360
Bored	−0.33	(0.14)	0.02*	360
Frustrated	−0.21	(0.14)	0.11	360
Challenged	0.01	(0.13)	0.97	360

Standardized scores are reported to allow for comparison across measures. The pretest standardized scores were calculated using the average grade standard deviation from Grade 4 and Grade 5. The posttest standardized scores were calculated using the average grade standard deviation but from the control group only. Positive emotion composite is the average standardized scores of Happy, Excited, and Hopeful items. Negative emotion composite is the average standardized scores of Bored, Nervous, and Frustrated items. The covariates include the respective standardized pretest tomoji score and grade level. Clustered standard errors by teacher are in parentheses. * $p < .05$.

Table A6 Estimated treatment by moderator effects of student gender and grade level.

Predictor Variables	*N*	*b*	*(SE)*	*p*
Gender Model				
Positive Emotion Composite				
Female	298	−0.11	(0.13)	0.40
Treatment	298	0.35	(0.15)	0.03*
Female X Treatment	298	−0.11	(0.18)	0.53
4th grade	298	0.05	(0.11)	0.68
Pretest positive emotion composite	298	0.59	(0.05)	0.00***
Negative Emotion Composite				
Female	299	−0.00	(0.09)	0.99
Treatment	299	−0.34	(0.10)	0.00***
Female X Treatment	299	0.21	(0.13)	0.11
4th grade	299	0.05	(0.10)	0.61
Pretest negative emotion composite	299	0.59	(0.05)	0.00***
Grade Model				
Positive Emotion Composite				
4th grade	313	0.00	(0.16)	0.99
Treatment	313	0.23	(0.14)	0.12
4th Grade X Treatment	313	0.08	(0.12)	0.25
Pretest positive emotion composite	313	0.60	(0.05)	0.00***
Negative Emotion Composite				
4th grade	314	0.13	(0.16)	0.44
Treatment	314	−0.13	(0.18)	0.47
4th Grade X Treatment	314	−0.11	(0.22)	0.62
Pretest negative emotion composite	314	0.61	(0.05)	0.00***

The results are based on two separate models with posttest nervousness and negative emotion composite as outcomes. The outcomes are regressed on respective interaction terms and main effects (treatment X pretest nervousness and treatment X pretest negative emotion composite) and grade level. Standardized scores are reported to allow for comparison across measures. The pretest standardized scores were calculated using the average grade standard deviation from Grade 4 and Grade 5. The posttest standardized scores were calculated using the average grade standard deviation but from the control group only. Positive emotion composite is the average standardized scores of Happy, Excited, and Hopeful items. Negative emotion composite is the average standardized scores of Bored, Nervous, and Frustrated items. Clustered standard errors by teacher are in parentheses. * $p < .0$, *** $p < .001$.

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