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Fraction Ball impact on student and teacher math talk and behavior

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ABSTRACT

We assessed the impacts of Fraction Ball—a novel suite of games combining the benefits of embodied guided play for math learning—on the math language production and behavior of students and teachers. In the Pilot Experiment, 69 fifth and sixth graders were randomly assigned to play four different Fraction Ball games or attend normal physical education class. The Efficacy Experiment was implemented to test improvements made through co-design with teachers with 160 fourth through sixth graders. Researchers observed and coded for use of math language and behavior. Playing Fraction Ball resulted in consistent increases of students' and teachers' use of fraction (*SDs* = 0.98–2.42) and decimal (*SDs* = 0.65–1.64) language and number line arithmetic, but not in whole number, spatial language, counting, instructional gesturing, questioning, and planning. We present evidence of the math language production in physical education and value added by Fraction Ball to support rational number language and arithmetic through group collaboration.

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Introduction

Proficiency in mathematics functions as a gatekeeper to advancement in academics and higher-paying careers in the United States (National Mathematics Advisory Panel, 2008). Despite a wealth of research around rational number understanding (Carpenter et al., 2012; DeWolf et al., 2015; Lewis et al., 2015; Siegler & Lortie-Forgues, 2014) and increasing efforts to reform instructional routines around teaching rational number understanding during the past four decades (e.g., Common Core State Standards initiative; Kober & Rentner, 2012), progress has been slow, with negligible differences in the proportion of students able to solve fraction and/or decimal arithmetic problems (Lortie-Forgues et al., 2015). Many have begun to investigate how to support children's academic success through playful enrichment activities (Bustamante et al., 2020; Hassinger-Das, Bustamante, Hirsh-Pasek, & Golinkoff, 2018; Ramani, Daubert, & Scalise, 2019; Ramani & Siegler, 2008; Yu et al., 2018). Such activities enhance conversations between caregivers and children to include academic language around mathematics (Purpura et al., 2020; Zhang et al., 2020).

The focus on math talk and language stems from a rich literature in the science of learning, demonstrating longitudinal associations between math language and math outcomes (King & Purpura, 2021; LeFevre et al., 2010). Math vocabulary accompanies learning about abstract math concepts, which can be conducive to improved mathematical competencies (Purpura et al., 2019). In addition to math language, playful enrichment activities provide additional learning opportunities through multisensory cues and the embodiment of mathematical concepts that help children to connect concrete experiences with abstract mathematical concepts (Nathan, 2021; Ramani, Daubert, & Scalise, 2019; Siegler & Ramani, 2009). Board games, for example, provide redundant multisensory cues about numerical magnitudes through the visual display of ordered numbers, the kinesthetic movement of the game pieces, and the auditory feedback of counting out loud as game pieces are moved (Jordan et al., 2008). Building on this work, the theory of embodiment emphasizes how physical movement and sensory experiences support conceptual learning (Barsalou, 2008; Nathan, 2021; Wilson, 2002). For example, the performance of walking a number line helps to ground the concept of numerical magnitudes by connecting children to a physical experience that gives that concept meaning; moving farther indicates a numerical magnitude increase. Indeed, when children walk along a number line, they demonstrate improved understanding of the spatial magnitude representation of whole numbers (Dackermann et al., 2017; Link et al., 2013). Altogether, the literature demonstrates the potential of interventions focusing on improving mathematics concepts through math language (Purpura et al., 2017), multisensory cues (Jordan et al., 2008), and embodiment (Nathan, 2021) when they are directly connected to the mathematical concepts being learned.

Fraction Ball is a collection of six games based in math cognition design principles by researchers and teachers to promote children's math language, related behaviors, and thinking about rational numbers (fractions and decimals) through a physically active and playful learning context (see Fig. 1 and Alvarez-Vargas et al., 2023, for further review). Fraction Ball games are played on a basketball court where the traditional 3-point line is redrawn to reflect a 1-point line split into arches reflecting parts of the whole area split into either thirds or fourths. The games are designed for math teachers to guide student teams as they compete against each other in games with different rules and objectives. For example, in "rapid fire" the objective is for a team to achieve the highest score, but in "exactly" the objective is for teams to reach a specific value first without surpassing it. Students play in teams of four and take on integral roles; the shooter takes shots on the redesigned court, the rebounder calls out the value of the shot from where it is made, and the counters use a life-size number line painted on the sidelines of the court as a scoreboard.

A previously published evaluation of Fraction Ball (Bustamante et al., 2022) demonstrated that this intervention increased students' rational number understanding, specifically their ability to convert fractions to decimals. In the current study, we evaluated how Fraction Ball affected student and teacher math language production and engagement in math-related behaviors, both of which were intermediate targets of the intervention. We estimated intervention impacts on language and behaviors because (a) math language supports children's conceptual mapping between the symbols and the magnitudes and relationships that abstract mathematical symbols represent (Moses & Cobb, 2002),

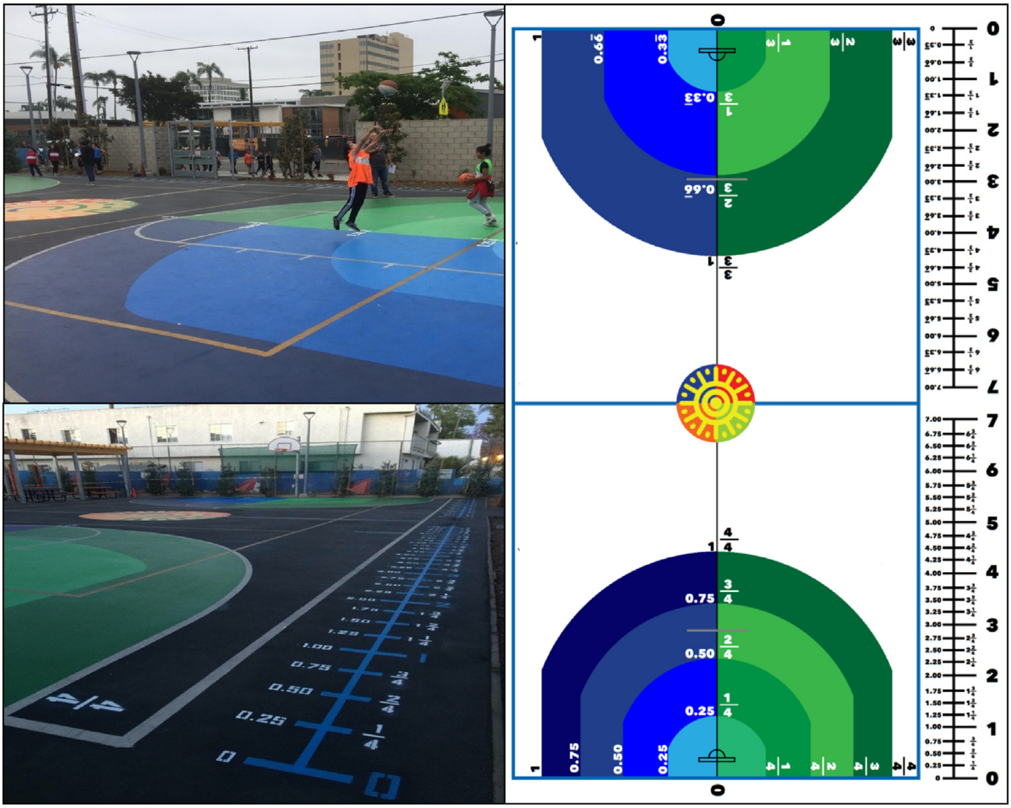

Fig. 1. Fraction Ball court. Right: A number line to assist children with keeping track of their score ranging from 0 to 7, with fractions on the right side and decimals on the left side of the court. The Fraction Ball regions reflect a basketball court with the traditional 3-point arch redrawn as a 1-point arch split into fourths on one end and thirds on the other end of the court. Each end court is split in half directly in front of the basket so that one-half of the arch reflects the quantity in fraction notation and the other half of the arch reflects the quantity in decimal notation. Left: Picture showing children playing in the shooter, rebounder, and counter roles. Students play on separate teams of four to compete for the highest number of points or to be the first to get to a specific number of points.

(b) the amount of mathematics language that children hear in informal learning environments predicts later mathematics outcomes (Berkowitz et al., 2015; Purpura et al., 2019; Ramani et al., 2015; Susperreguy & Davis-Kean, 2016), and (c) embodied behaviors, such as students' and teachers' mathematical expressions through instructional gesture, demonstrate math knowledge and support math learning (Gordon & Ramani, 2021).

Fraction Ball as a playful learning landscape

Play provides a socially interactive context for promoting children's learning and development through an active, meaningful, and engaged setting (Hirsh-Pasek et al., 2015; Schlesinger et al., 2020; Zosh et al., 2018) that is low stakes and motivating (Maloney & Beilock, 2012). Such activities can be designed to support participation structures that provide students with access to the domain, integral roles, and opportunities for self-expression to enhance deep engagement (Nasir & Hand, 2008). Moreover, when activity structures incorporate whole body learning experiences, they produce greater learning gains than passive contexts, especially for younger children (Dackermann et al., 2017). Considering these principles of learning design, Playful Learning Landscapes (PLL) is a promising initiative that combines that value of play in promoting engagement and whole body learning experiences that increase science, technology, engineering, and mathematics (STEM)-based language and informal conversations (Bustamante et al., 2020; Hassinger-Das, Zosh, et al., 2020; Schlesinger et al., 2020).

Multiple Playful Learning Landscapes have been successfully designed to promote STEM language, child–caregiver interactions, and questioning. Some examples, include the "Supermarket Speak" study, where colorful signs were installed in grocery stores to increase learning opportunities for children through caregiver–child interactions such as conversations about where milk comes from (Hassinger-Das et al., 2018; Ridge et al., 2015). This study found an increase of 33% in child–caregiver interactions in supermarkets in low-income neighborhoods. In "Parkopolis," a life-size board game in a museum (Bustamante et al., 2020), children and caregivers were observed to use more STEM language, engagement, interaction, physical activity, and questioning in comparison with a STEM museum exhibit (Gaudreau et al., 2021). In "Urban Thinkscapes," where children engage with spatial puzzles, executive function hopscotch, hidden figures, and stories in a transformed bus stop (Hassinger-Das, Palti, et al., 2020), children showed increased interactions with caregivers and conversational turns of numerical and spatial language. These studies have provided the foundation for an ongoing global initiative that seeks to transform public spaces and everyday experiences to infuse them with playful learning opportunities that promote STEM language and skills (Schlesinger et al., 2020).

Infusing the science of learning in the design of Fraction Ball

Fraction Ball is a redesigned game of basketball that brings the PLL initiative to the school yard space where teacher-guided games provide opportunities for children to take on integral roles as they use math language and gestures to collaboratively solve math problems invoking fraction and decimal number reasoning. Fraction Ball draws from the following design principles in educational research to support students' math learning: number line for integrating whole numbers and rational numbers (Siegler & Lortie-Forgues, 2014), learning through comparisons (Begolli & Richland, 2016; Booth et al., 2017; Rittle-Johnson et al., 2020), embodied cognition (Dackermann et al., 2017; Link et al., 2013; Ramani & Siegler, 2008), and challenging misconceptions around fraction and decimal notations (Durkin & Rittle-Johnson, 2012; Hiebert & Wearne, 1985; Lortie-Forgues et al., 2015). The theory of Fraction Ball and its impacts on mathematics performance are discussed at greater length in Bustamante et al. (2022). Here, we focused on how Fraction Ball game rules and design (Fig. 1) provide affordances to produce rational number language and math reasoning behavior. In the current study, our goal was to examine the effects of playing Fraction Ball during physical education (PE) class compared with regular PE class on students' and teachers' production of mathematical language and behavior.

The number line. The Fraction Ball court design and game rules are based on the integrated theory of numerical development (Siegler et al., 2011), which postulates that a mental number line representation is foundational for integrating children's knowledge of whole numbers and rational numbers as magnitudes shown to improve performance on multiple rational number tasks. The Fraction Ball court reinforces rational number magnitude representations by relabeling the 3-point arc into a 1-point arc, with additional arcs closer to the basket labeled as fourths or thirds (Fig. 1) such that the spatial distance corresponds to the magnitude of the fraction/decimal marking the inner arcs. The arcs are designed to reflect number lines that radiate out from the basket. As students move farther away from the basket, the magnitude of the shot will always be greater than the magnitudes of shots made closer to the basket. This decision was influenced by the representational mapping hypothesis that was tested by Siegler and Ramani (2009), who demonstrated that it is easier for students to learn and internalize linear representations of numerical magnitudes when the numerical magnitudes are physically presented to them in a linear fashion. We made explicit design decisions to ensure that the court represented rational numbers as representative of magnitude gains and not as discrete or categorical numbers. One example was the shape of the arcs. The use of arcs in which the whole arc had the same rational number label precluded placement of the smallest number next to the largest number, which can occur in circular board games and may mislead students about the relationship between number and magnitude. We also juxtaposed the court with the linear number lines on the sidelines to more accurately reflect the continuous nature of the incremental scores made from the arcs on the court. The common goal of the games is gaining scores higher than 1, but this can be difficult to map on the arcs because each one denotes a number less than or equal to 1 and a team's score reflects a composite of multiple shots. Therefore, students must build on the Fraction Ball court representation by plotting their scores on a linear number line. In addition, there are redundant multisensory cues (Ramani, Daubert, & Scalise, 2019) to magnitude throughout the game, such that if a student is farther away from the basket the value of a made shot increases proportionally to when the student is close to the basket; the shot is also harder to make, and the court arch color gets darker as well.

The Fraction Ball court has two number lines on the sidelines that are used as scoreboards. Players keep score through arithmetic (e.g., adding the last shot made to the current score) by moving a bean bag along the number line (Fig. 1). While observing the first Pilot Experiment, students would yell "25" rather than "0.25" when playing the games. So, to reinforce rational number language that connects to the mental and physical representations of the number line, we introduced a rule where children are required to state their rational number arithmetic sentence out loud (e.g., one-fourth plus three-fourths equals four-fourths) prior to moving along the number line (e.g., moving from 1/4 to 4/4). This was inspired by Siegler and Ramani's (2009) number line experiment where students were required to count out loud beginning from the previous number such that a child beginning at the number 3 would add 2 more by counting up, saying "4, 5" rather than "3, 1, 2".

Comparing and contrasting. The court arches and number lines are drawn to compare the fractional and decimal representations of magnitudes to help students draw connections between language and notations using the cognitive principles of analogies and/or comparing and contrasting representations (Richland & Begolli, 2016; Richland et al., 2017; Richland & Simms, 2015; Rittle-Johnson et al., 2020). Language comparisons can serve to highlight the commonalities and differences between surface features (e.g., stating zero point twenty-five and one-fourth) and structural features (e.g., both 0.25 and 1/4 represent the same magnitude on the number line). Moss and Case (1999) and Kalchman and colleagues (2001) showed that with a curriculum in which fraction and decimal notations and language were used interchangeably, children improved their knowledge of rational numbers on multiple indices. Because children commonly treat fraction and decimal notations as representing separate mathematical systems, we followed the examples of other researchers (Hiebert & Wearne, 1985; Vamvakoussi & Vosniadou, 2010; Wang & Siegler, 2013) in promoting the relationship between these two language and symbol systems.

The rules of the Fraction Ball games require teams to either state their calculations out loud in fraction form *or* in decimal form for one round of play. After each round, teams switch sides (from fraction side to decimal side and vice versa) to gain practice with both symbolic and language representations. This design, in combination with the rule to state their arithmetic sentences and the magnitude of each point made, may afford children to notice that the differences in the fraction and decimal symbols and language are at the surface level and do not affect the magnitude representations or the equivalency between fractions and decimals.

Math language. Over the past decade, there has been increasing empirical support for the theoretical role of a linguistic pathway on the development of mathematical competence (LeFevre et al., 2010; Levine et al., 2010; Purpura et al., 2019). Mathematical language is hypothesized to function (a) as a medium used to represent and communicate mathematical reasoning and (b) as the representation of mathematical thinking (Peng et al., 2020). Specifically, mathematical vocabulary may serve as the format through which arithmetical facts can be stored in and retrieved from memory; however, the influence of different forms of math vocabulary (e.g., cardinal numbers, spatial words) on different mathematical outcomes (e.g., magnitude comparison, word problems) varies (Peng & Lin, 2019).

Different subtypes of math language differentially predict children's mathematical outcomes. For example, parents' number talk such as counting and labeling of visible objects predicts their toddler's cardinal number knowledge (Gunderson & Levine, 2011; Levine et al., 2010). Moreover, the associations between language skills and math skills grow in strength across time as mathematical skills grow in sophistication (Chow & Jacobs, 2016). Exposure to mathematical language is a consistent unique predictor of fourth graders' conceptual understanding of fractions, which supports the hypothesis that language plays an additional key role in how children learn fraction concepts (Jordan et al., 2013). Both correlation and experimental work have shown the important role that math talk and language play in the development of children's mathematical knowledge (King & Purpura, 2021; Purpura et al., 2019; Toll & Van Luit, 2014a, 2014b).These associations support the pathways model, which states that a linguistic pathway supports the symbolic numerical system through which mathematical learning objectives like magnitude comparison occur (LeFevre et al., 2010).

Playful Learning Landscapes have been found to elicit significant increases in diverse types of math language interactions between caregivers/teachers and children that are predictive of math outcomes (Eason & Ramani, 2018; Fisher et al., 2013; Hassinger-Das et al., 2018; Ramani & Scalise, 2020; Schlesinger et al., 2020), although many of these studies have not yet been able to causally assess the impact of these learning opportunities on children's math skills. A few randomized control trials (RCTs) reveal the positive impact of math language on mathematical performance (Hassinger-Das et al., 2015; Jennings et al., 1992; Powell & Driver, 2015; Purpura et al., 2017). Yet, most of this work has centered around the impact of language on whole numbers and calculations and rarely around more complex mathematical concepts (Peng et al., 2020).

Fraction Ball games are learner-centered and teacher-guided to purposefully elicit language describing the manipulation of fractions and decimals; thus, we may find that the games provide the opportune mathematical language intervention for rational number reasoning because this rational number language is less likely to naturally occur in free-play settings such as those examined in previous PLL evaluations. The complexity of mathematical discussions that were observed to have been elicited by the games included different forms. Math talk was evident in the direct use of numbers to represent quantities and conduct calculations such as "one-third," "two-fourths," and "three" (Levine et al., 2010). Math language was evident in content-specific discussions about quantitative features such as "how many," "less than", and "more than" (Purpura & Logan, 2015). Spatial language was verbalized regarding relational movements and court features such as "take a shot from farther away" and "shoot above your head" (Cannon et al., 2007; Pruden & Levine, 2017). The use of rational number language is reinforced by Fraction Ball game rules requiring players to state the magnitude of each basket made in fraction/decimal form out loud in addition to explaining their math sentences when adding up scores (encouraging number/math talk; Levine et al., 2010) as they take shots from different distances on the court and develop game strategies in teams (encouraging math language and spatial language). This serves to promote the connection between rational number language use and the quantitative magnitude understanding through an embodied spatial representation that is paired with their physical movements on the court.

Math talk and language play a particularly important role in our partner school, which is dual immersion (school instruction begins fully in Spanish in kindergarten and gradually phases out across grade levels), and most of the students are English language learners (ELLs). Many of our partner teachers were once students who learned English as a second language and now teach ELLs mathematics. Teachers explained that as Common Core math moved from being predominantly about procedures and skills to being about conceptual understanding and sense making, it has also shifted toward word problems that require at-level reading and comprehension. Their concerns were that students who are learning in a dual-immersion program might not have math vocabulary in English that is as robust as their English-dominant counterparts. This creates a unique situation where teachers try to bridge the Spanish version of the math vocabulary to English and sometimes use synonyms rather than standard math vocabulary. They observed that although students may know the math, their understanding of standard math vocabulary, and of English reading levels in middle school, inhibits their production of accurate math strategies. For example, one teacher observed that throughout the 5 years she worked in middle school, her students were able to compute an average, but when presented with a word problem asking for the "mean," students were unaware of what this meant. Therefore, in our current study context, the exposure to standard math vocabulary and embodied rational number reasoning tasks may help students to build connections in the math vocabulary that represents these mathematical procedures.

The current study

In this study, we estimated the experimental impacts of Fraction Ball on math talk and related behaviors in two previous experiments found to boost students' rational number knowledge (Bustamante et al., 2022). The Fraction Ball games were designed to elicit the use of specific math vocabulary and behaviors that are directly connected to the math concepts being learned to bolster mathematical understanding. We contrast the math language and related behaviors used in the Fraction Ball games with the same outcomes in children randomized to the control group, who participated in a typical PE class. We assessed math language, including numerical values (e.g., whole numbers, fractions, decimals; Klibanoff et al., 2006), counting (e.g., one, two), spatial vocabulary (e.g., closer, above, below; Cannon et al., 2007), and grouping and comparing (e.g., more than, fewer than; Gunderson et al., 2015). In addition, we assessed math-related behaviors such as conducting arithmetic, questioning, planning, and instructional gesturing (e.g., pointing to the number line; Gordon & Ramani, 2021).

We expected to see increases in the amount of math language and related behaviors for students and teachers because the games incorporate rules that require students to manipulate rational number magnitudes and scripts that require teachers to guide game play and provide scaffolding. Although math language and related behaviors are likely not the only mechanisms through which the Fraction Ball intervention improved students' rational number knowledge, we used these measures as proxies to get a sense of where the language impacts were largest and where they may have been absent. These differences in impacts can characterize how much various kinds of language changed during the intervention, which will be useful for understanding key design components that differentiate the intervention from regular PE.

Hypotheses

Based on the language that is directly encouraged in the Fraction Ball games script, we predicted that the largest treatment effects on the verbalizations used by both the students and teachers would be on whole numbers, fractions, and decimal words. We also hypothesized smaller treatment effects that range from largest to smallest based on the behaviors directly encouraged by the intervention. Therefore, we expected greater impacts on number line arithmetic and mental/body arithmetic than on math instructional gesture, planning, grouping and comparison, noticing patterns, questioning, counting, and spatial talk. We hypothesized larger impacts on number line arithmetic than on mental/body arithmetic because of the rules requiring students on the number line to explain their mathematical reasoning out loud. Because students were in the process of building foundational connections between rational number magnitudes and rational number arithmetic, we expected them to better offload the working memory resources on the number line as opposed to mental/body strategies such as counting fingers.

Method

Participants

In the Pilot Experiment, four teachers volunteered to participate in the experiment: one fifth-grade teacher, one STEM coordinator, and two sixth-grade teachers. A total of 69 children (35 fifth graders and 34 sixth graders) from two fifth-grade and two sixth-grade classrooms gave assent to enroll in the study. Children were predominantly Latine (>90% in the school) and 45% female. In all participating classrooms, half of the students ($n = 16$) within the same classroom were matched on gender and standardized pretest score and then were randomly assigned to participate in the Fraction Ball intervention during PE, and the other half remained in their usual PE.

In the Efficacy Experiment, 10 teachers participated in conducting six games with 16 students from each of their classrooms: four fourth-grade, four fifth-grade, and two sixth-grade classrooms. Assent forms were handed out to 232 students, 195 of whom assented to participating in the Fraction Ball games; only 160 students were randomly assigned to provide equal treatment groups within classrooms. This randomization yielded a total of 8 children (4 boys and 4 girls) for each treatment condition ($8 \times 2 = 16$) within each class (16×10), making our full analytical sample 160 students. Due to involvement in other school activities, 3 students did not comply with being assigned to receive the Fraction Ball intervention; however, these students were still observed in the control condition and were included in the analysis sample to estimate the effect of being assigned to the treatment. Statistical equivalence values for all measured child-level variables across treatment groups are provided split by treatment group in Tables 1 and 2.

All study procedures were approved by the University of California, Irvine, institutional review board.

Procedure

The Fraction Ball intervention was designed to improve children's rational number reasoning and knowledge via math language and math-related behaviors. The first experiment was a pilot study to determine the feasibility of implementing the Fraction Ball games, which consisted of a set of four games that were played in a 50-min PE class period. The games are explained in a script that teachers followed as they facilitated the games; the full Pilot Experiment script is shown in Appendix A in the online supplementary material. The script details the rules of the game that are intentionally designed to reinforce rational number learning, such as a rule that the rebounder calls out the points made when the shooter makes a shot (e.g., "one-fourth" or "0.25") and the counters on the number line must explain their reasoning out loud to move ahead (e.g., "0.75 + 0.25 = 1").

Three participating teachers co-designed the Fraction Ball court and script with researchers during three meetings in the spring of 2019. The co-design approach ensured that the math content aligned with their classroom instruction and their students' math knowledge. The games have different objectives, such as making as many points as possible in 2 min or racing to an exact number (e.g., 3.25) without overshooting the goal. For Post Pilot Experiment 1, teachers and researchers met in the fall of 2019 to make improvements to the Fraction Ball script and evaluate the effectiveness of Fraction Ball in a larger sample.

In Efficacy Experiment 2, the Fraction Ball script included two additional games called "Make It Count Ghost" and "Exactly Flip"; the full script is available in Appendix B in the supplementary material. In "Make It Count Ghost," students were required to engage in more rational number estimation by needing to decide the position of their score on a number line from 0 to 5 with no tick marks in between, and the team with the most accurate estimation won. In Experiment 1, we found that students had not improved their ability to add fractions and decimals. To address this, the researchers designed "Exactly Flip," which required students to randomly flip from the decimals side of the court to the fractions side of the court or vice versa. This required students to add their scores using fractions and decimals frequently throughout the game.

To increase students' arithmetic reasoning with different types of fractions, two new game rules called "who shoots first" and "super charge points" were also incorporated. The "who shoots first" game rule meant that teachers would call out a number, such as an improper fraction like 5/4 or 6/3, and both teams would need to figure out and jump to where they thought the number is placed on the number line. The student who got to the correct number first was able to take the first shot of the game. The "super charge points" game rule allowed students to make extra points when the teacher randomly called rational number values from a predetermined list. The two student counters would have the responsibility of adding the points to their current score, and if their calculations were correct, their team would get the points. Altogether, these design changes reflected the goals of increasing students' rational number reasoning via talking and thinking about rational number magnitudes.

Fraction Ball court

As shown in Fig. 1, the distance from below the basket to the 3-point arc is converted into a "0 to 1" area with arcs that act as number line markers and divide one end of the court into fourths and the other end into thirds. Arcs closer to the basket represent 1/4-, 1/2, and 3/4-point shots on one end

Table 1
Statistical balance tables of participant demographics and missingness for both experiments

Variable	Full sample			Control			Treatment			p
	n	M	SD	n	M	SD	n	M	SD	
Pilot Study										
Compliance	69	80%		37	78%		32	81%		.77
Male	69	55%		37	51%		32	59%		.51
Fifth grade	69	51%		37	51%		32	50%		.91
Sixth grade	69	49%		37	49%		32	50%		.91
Teacher rating: Low	69	30%		37	27%		32	34%		.52
Teacher rating: Average	69	30%		37	30%		32	31%		.89
Teacher rating: High	69	39%		37	43%		32	34%		.46
Dosage in days							32	3.59	0.65	
Attrition	69	1%		37	3%		32	0%		.36
Efficacy Study										
Compliance	160	98%		83	96%		77	100%		.09
Male	160	49%		83	46%		77	52%		.44
Fourth grade	160	41%		83	41%		77	42%		.94
Fifth grade	160	40%		83	40%		77	40%		.95
Sixth grade	160	19%		83	19%		77	18%		.86
Dosage in days							77	5.92	0.31	

Note. No significant p-values demonstrated baseline balance on all observable characteristics, indicating appropriate randomization. Attrition occurs when students complete the pretest but do not complete the posttest. Teacher ratings were teachers' self-reported rankings of students' math proficiency in their classrooms; levels were preset by teachers as being below average, average, or above average. These data were collected to ensure randomization of students based on teacher perception.

of the court and 1/3- and 2/3-point shots on the opposite end (Fig. 1). To promote fraction to decimal conversions, each end of the court was split in half with fraction and decimal symbols side by side (e.g., 1/4 [0.25], 2/3 [0.66]). On the sideline of the court, children tallied their scores on a 0–7 number line with fraction and decimal representations (Fig. 1). During the games, children took on three roles: shooter, rebounder, and counter. During the games, the shooter takes a shot, the rebounder calls out the value of the shot, and the counter moves down the number line to tally the score.

Student randomization was conducted after pretest scores were collected to stratify by classroom, gender, and pretest rational number skills to ensure comparable groups. Fraction Ball sessions took place twice a week for 2 weeks and were run by two teachers (one on each end of the court), with each session lasting 35 to 40 min. All teachers attended a professional development session guided by the research team to practice conducting the games. Teachers were instructed to follow the script to ensure that students played the games in the same order and had a similar experience across groups. In addition, teachers were instructed to not share the script with other teachers or play the games with other students until the end of the intervention evaluation to prevent crossover effects. Naturalistic observations were conducted by trained researchers to capture student and teacher language and behavior across conditions.

Fidelity of implementation

To capture the extent to which teachers administered the intervention as intended, we observed that 66% of the intervention sessions used a fidelity rubric. Trained observers marked each of the activities and recorded the duration of each activity with a stopwatch. Of these observations, 13% were double-coded, and all coders established greater than 80% interrater reliability prior to coding independently.

Observer training

Three authors from the research team and 10 undergraduate research assistants conducted live observations of language utterances during the Fraction Ball game sessions. Each observer completed a 1-hr training workshop and conducted a live double-coding observation with one of the lead authors until reliability was established. As a final check for reliability, 25% of total observations were double-

Table 2
Average utterances across for each cycle observed split by treatment groups and study

	Pilot Study							Efficacy Study						
	Control			Treatment				Control			Treatment			
	n	*M*	*SD*	*n*	*M*	*SD*	*p*	*n*	*M*	*SD*	*n*	*M*	*SD*	*p*
Students														
Whole numbers	54	1.50	0.67	133	1.97	0.68	<.001	52	1.95	1.10	108	2.26	0.78	.045
Fractions	54	1.02	0.14	133	2.25	0.75	<.001	52	1.00	0.00	108	2.54	1.05	<.001
Decimals	54	1.02	0.14	133	1.77	0.69	<.001	52	1.00	0.00	108	1.82	0.87	<.001
NL arithmetic	54	1.00	0.00	133	1.50	0.61	<.001	52	1.02	0.14	108	1.94	0.78	<.001
Mental/Body arithmetic	54	1.00	0.00	133	1.20	0.42	.001	52	1.00	0.00	108	1.31	0.44	<.001
Counting	54	1.14	0.34	133	1.10	0.29	.407	52	1.29	0.67	108	1.19	0.47	.281
Patterns	54	1.00	0.00	133	1.07	0.25	.050	52	1.02	0.14	108	1.08	0.26	.098
Spatial	54	1.55	0.55	133	1.35	0.52	.019	52	1.55	0.69	108	1.30	0.44	.007
Instructional gesture	54	1.61	0.62	133	1.25	0.42	<.001	52	1.02	0.14	108	1.18	0.43	.012
Grouping and comparing	54	1.05	0.20	133	1.18	0.37	.015	52	1.17	0.43	108	1.26	0.43	.216
Questioning	54	1.32	0.45	133	1.41	0.50	.272	52	1.35	0.48	108	1.54	0.47	.018
Planning	54	1.51	0.49	133	1.58	0.62	.439	52	1.39	0.53	108	1.53	0.60	.174
Teachers														
Whole numbers	54	1.71	1.08	133	2.25	0.87	<.001	52	1.84	0.87	108	2.35	0.95	.001
Fractions	54	1.00	0.00	133	1.85	0.63	<.001	52	1.04	0.19	108	1.97	0.75	<.001
Decimals	54	1.01	0.07	133	1.49	0.58	<.001	52	1.02	0.14	108	1.37	0.53	<.001
NL arithmetic	–	–	–	–	–	–		52	1.00	0.00	108	1.19	0.37	.004
Mental/Body arithmetic	9	1.00	0.00	26	1.08	0.27	.406	52	1.00	0.00	108	1.12	0.29	.351
Counting	54	1.36	0.48	133	1.29	0.43	.318	52	1.17	0.39	108	1.25	0.49	.939
Patterns	54	1.02	0.10	133	1.05	0.21	.370	52	1.10	0.28	108	1.09	0.27	.062
Spatial	54	1.46	0.56	133	1.41	0.50	.553	52	1.70	0.58	108	1.53	0.53	.807
Instructional gesture	54	1.51	0.50	133	1.44	0.52	.379	52	1.15	0.54	108	1.17	0.36	.696
Grouping and comparing	54	1.07	0.25	133	1.53	0.59	<.001	52	1.36	0.54	108	1.39	0.48	.000
Questioning	54	1.41	0.48	133	1.94	0.65	<.001	52	1.29	0.53	108	2.17	0.72	.577
Planning	54	1.06	0.22	133	1.30	0.47	.001	52	1.18	0.43	108	1.22	0.41	<.001

Note. Pilot Study total cycles observed n = 187; Efficacy Study total cycles observed n = 160. NL, number line.

coded. There was one instance where data for a double-coded observation were lost (i.e., physical observation notes were lost prior to scanning); in this case, the field observation notes that were double-coded for this session were used.

To ensure that math language variation was accurately captured in the context of a Spanish to English dual immersion school, all researchers involved in conducting live observations identified as Latine and either fluently spoke Spanish or understood Spanish. All researchers were instructed to code for math language regardless of the language in which it was spoken. However, a very small proportion of Spanish was used in the observed grade levels because most of the in-classroom instruction from fourth to sixth grades begins to phase out from using Spanish to using predominantly English.

Fraction Ball observers stood on the sidelines of the basketball court and control condition observers stood alongside a chain link fence as students engaged in PE activities. For each Fraction Ball session, two researchers were randomly assigned to observe one condition, such that observations of both conditions could take place at the same time. On days when only one researcher could attend an observation session, we counterbalanced the order of observations so that half of the PE activities and half of the Fraction Ball games were recorded. Researchers recorded teacher and student language utterances and interactions in 3-min intervals using the observational protocol (Appendix B) until the end of the class period. The amounts of completed observation sessions across experiments and treatment are shown in Table 3.

The observational coding scheme and protocols (Appendix B) were adapted from a previous PLL study on the Parkopolis installation (Bustamante et al., 2020). Researchers tallied teacher and student

Table 3
Descriptive statistics of observations across experiments and treatment groups

Variable	Pilot Study		Efficacy Study	
	Control	Treatment	Control	Treatment
Sessions observed	11	22	12	17
Average cycles within sessions: *M* (*SD*)	3.35 (1.95)	4.45 (2.46)	3.88 (2.52)	4.17 (2.38)
Cycles observed by teacher	T0 (54)	T1 (87) T2 (41) T4 (5)	T0 (52)	T1 (39) T2 (8) T3 (15) T5 (3) T6 (33) T7 (10)

Note. T1 and T2 in Experiments 1 and 2 are the same teachers. T0 is the reference physical education teacher. The teacher was not always the same across observations.

language and behaviors, including whole number, fraction, decimal, arithmetic on the number line, arithmetic done with the body or in the mind, counting, patterns, spatial talk, instructional gesture, grouping and comparing, questioning, and planning. To achieve reliable and manageable codes, teacher and student language tallies were recoded on a 5-point scale (1 = 0 utterances, 2 = 1 to 5 utterances, 3 = 6 to 10 utterances, 4 = 11 to 15 utterances, and 5 = 16 or more utterances); exact ratings across multiple coders on the 5-point scale were considered reliable. Reliability across the recoded language and behavior categories mentioned above ranged from α = .60 to α = .80. We conducted sensitivity checks to assess the stability of our estimates when excluding observations from coders who did not reach at least α = .75 reliability.

Analysis plan

We preregistered our analyses at the Open Science Framework (https://osf.io/6qysh) after the Pilot Experiment (n = 69) and prior to data collection for the Efficacy Experiment (n = 160). In the preregistration, we proposed a multiple regression to estimate the treatment impact on language use for students and teachers while controlling for student grade and the teacher facilitating the activities [Equation (2)]. Specifically, we calculated a composite score averaging the frequencies of all language categories used by teachers and students that were observed during the same observation. We observed multiple cycles nested within multiple observations for each teacher directing a group of students.

Linear regression:

$$Y_i = \beta_0 + \beta_1 * \mathrm{Treatment}_i + e_i \tag{1}$$

Multiple linear regression (preregistered controls):

$$Y_i = \beta_0 + \beta_1 * \mathrm{Treatment}_i + \beta_2 * \mathrm{Grade}_i + e_i, \tag{2}$$

where Y_i is the value of the number of language utterances for the ith observation, Treatment is a dichotomous variable where 1 = Fraction Ball, Grade is a vector of grade level dichotomous variables with fourth grade as the reference, and e is a random error. We deviated from our preregistered analysis plan by excluding teacher fixed effects because teachers were perfectly correlated with treatment assignment. We did not anticipate this constraint prior to implementing the study and checked the robustness of this analytical decision by comparing our fixed effect estimates with random effect estimates as shown in Tables S7 and S8 in the supplementary material, which demonstrate that our results were consistent.

Results

Before proceeding with analyses, we checked for imbalance in the number of observations and the duration of observations across the treatment groups. We counterbalanced observers across treatment groups, such that two observers captured activity at opposite ends of the basketball court and one observer captured the control group in the PE area. Researchers collected more observations of the treatment group (22 in the Pilot Experiment and 17 in the Efficacy Experiment) than of the control group (11 in the Pilot Experiment and 12 in the Efficacy Experiment). This observational protocol was planned to oversample the experimental lessons to better understand the impact of different game rules at the different ends of the court (i.e., fourths and thirds half-courts). In addition, the control group observer often observed whole group activities directed by the PE coaches; therefore, variation in language was easier to capture with one observer. In the Pilot Experiment, there were on average 3.35 (SD = 1.95) cycles observed for the control group and 4.45 (SD = 2.46) cycles observed for the treatment group in each of these sessions; group differences were not significant, $t(31) = 1.29$, $p = 0.206$. For the Efficacy Experiment, the average cycles observed in each session were 3.88 (SD = 2.52) for the control group and 4.17 (SD = 2.38) for the treatment group; group differences were not significant, $t(27) = 0.334$, $p = .744$.

Will playing Fraction Ball affect students' and teachers' math language production?

First, we discuss the impacts on student language production, all of which are shown in Table 4 split by experiment and regression specification. The first two columns of Table 4 show impacts estimated from the bivariate regression shown in Equation (1), and the last two columns show impacts estimated from Equation (2). We show both specifications to test the sensitivity of our estimates to variations in regression specification; however, readers should note that Equation (2) is our preferred preregistered specification. Participant demographics (i.e., gender and grade) were balanced across both treatment conditions. Estimates of treatment impacts on student language and behavior were larger in magnitude for the Efficacy Experiment and relatively consistent in size and direction across the model specifications.

Results were consistent across both experiments and grade levels; students produced more fraction words (βs = 0.74 to 1.44, ps < .001, ≈70–115 more words per class period) and decimal words (βs = 0.64 to 0.84, ps < .001, ≈ 51–67 more words per class period). In practical terms, these impacts reflect an increase of 1 to 5 more utterances of fraction and decimal words every 3 min; thus, in a 50-min period with approximately 16 3-min cycles, increases of 1 to 5 more utterances indicate an increased use of 16 to 80 more words per class period. This is a significant increase in rational number language considering the average of 0.04 to 0.24 words used by students and the average of 0.03 to 0.42 words used by teachers in the control group—meaning that in one PE period the teachers very rarely used rational number words, a rare learning goal in the PE curriculum.

Contrary to our hypotheses, we found a significant increase only in the use of whole number language in the Efficacy Experiment. Interestingly, there was a substantial amount of whole number language used in PE activities, ranging from about 1 to 5 utterances per 3-min cycle. Similarly, students in the PE condition demonstrated significantly greater spatial language (βs = −0.26 to −0.28, ps < .05, ≈ 20–22 more words per class period) than students in the Fraction Ball condition, but this was not consistent in the Efficacy Experiment. Lastly, the treatment impacts on the amount of counting, noticing patterns, and grouping and comparing were inconsistent across experiments and model specifications.

Next, we discuss the impacts on teacher language, all of which are shown in Table 5 following the same structure as Table 4. There were significant positive impacts on teachers' use of fraction and decimal language and their engagement with number line arithmetic. Specifically, the teachers' fraction language production (βs = 0.60 to 0.91, ps < .001, ≈ 48–72 more words per class period) and decimal language production (βs = 0.20 to 0.37, ps < .001, ≈ 16–29 more words per class period) increased in both experiments. There were also significant impacts on the amount of grouping and comparing that teachers did in Experiment 1 (βs = 0.40 to 0.44, ps < .001, ≈ 32–35 more words per class period); however, this did not hold across the different experiments and specifications.

Table 4
Fraction Ball impacts on observed student language and behavior across experiments

Dependent variable	Bivariate regression, no controls								Preregistered multiple linear regression with grade covariates							
	Pilot Study				Efficacy Study				Pilot Study				Efficacy Study			
	Control mean	*B*	*SE*	*p*	Control mean	*B*	*SE*	*p*	Control mean	*B*	*SE*	*p*	Control mean	*B*	*SE*	*p*
Language use																
Whole numbers	1.89	0.11	0.25	.65	1.59	0.63	0.17	<.001	1.89	0.26	0.25	.31	1.59	0.61	0.19	<.001
Fractions	1.13	0.88	0.16	<.001	1.00	1.39	0.20	<.001	1.13	0.74	0.15	<.001	1.00	1.44	0.24	<.001
Decimals	1.13	0.64	0.13	<.001	1.00	0.84	0.11	<.001	1.13	0.65	0.13	<.001	1.00	0.80	0.13	<.001
Counting	1.12	−0.05	0.05	.30	1.16	0.02	0.08	.75	1.12	−0.03	0.05	0.48	1.16	0.08	0.08	.34
Noticing patterns	1.03	0.02	0.05	.73	1.01	0.10	0.04	.02	1.03	0.04	0.05	.47	1.01	0.07	0.05	.14
Grouping and comparing	1.08	0.06	0.06	.27	1.09	0.14	0.08	.07	1.08	0.05	0.06	.39	1.09	0.18	0.09	.06
Spatial	1.56	−0.26	0.11	.02	1.46	−0.09	0.10	.39	1.56	−0.28	0.11	.02	1.46	−0.14	0.12	.23
Behavior																
NL arithmetic	1.00	0.33	0.11	<.001	1.01	0.78	0.10	<.001	1.00	0.30	0.12	.01	1.01	0.79	0.12	<.001
Mental/Body arithmetic	1.00	0.13	0.06	0.05	1.00	0.32	0.07	<.001	1.00	0.11	0.06	.10	1.00	0.32	0.09	<.001
Instructional gesture	1.48	−0.21	0.12	0.10	1.01	0.19	0.06	<.001	1.48	−0.25	0.13	.06	1.01	0.20	0.07	.01
Questioning	1.50	−0.13	0.11	0.25	1.29	0.20	0.10	.04	1.50	−0.11	0.12	.37	1.29	0.19	0.11	.09
Planning	1.59	−0.10	0.15	0.49	1.29	0.27	0.14	.05	1.59	−0.13	0.16	.43	1.29	0.17	0.16	.28

Note. Values reflect numbers of utterances or actions observed rescaled, such that 1 = 0 utterances, 2 = 1 to 5 utterances, 3 = 6 to 10 utterances, 4 = 11 to 15 utterances, 5 = 16 or more utterances. NL, number line. Covariates in preregistered model are students' grade level and their math teacher, where math teacher may or may not have been the teacher facilitating the Fraction Ball games. Observations: $N = 96$; Pilot Study $n = 43$; Efficacy Study 2 $n = 53$. The control means from the bivariate models (first from left) represent the regression model intercept. The control means are used to replace the control means for the preregistered multiple linear regression models because the control means in these models were lower than 1 due to the inclusion of grade level covariates.

Table 5
Fraction Ball impacts on observed teacher language and behavior across experiments

Dependent variable	Bivariate regression, no controls								Preregistered multiple linear regression with grade covariates							
	Pilot Study				Efficacy Study				Pilot Study				Efficacy Study			
	Control mean	*B*	*SE*	*p*	Control mean	*B*	*SE*	*p*	Control mean	*B*	*SE*	*p*	Control mean	*B*	*SE*	*p*
Language use																
Whole numbers	1.77	0.49	0.23	.04	2.09	0.20	0.18	.27	1.77	0.59	0.24	.02	2.09	0.31	0.18	.09
Fractions	1.00	0.68	0.12	.00	1.30	0.60	0.15	.00	1.00	0.58	0.12	.00	1.30	0.91	0.12	.00
Decimals	1.03	0.36	0.08	.00	1.18	0.20	0.11	.07	1.03	0.32	0.09	.00	1.18	0.37	0.10	.00
Counting	1.46	−0.11	0.11	.34	1.32	−0.10	0.10	.33	1.46	−0.03	0.11	.82	1.32	0.01	0.08	.94
Noticing patterns	1.11	−0.06	0.06	.36	1.27	−0.13	0.09	.13	1.11	−0.03	0.06	.61	1.27	−0.12	0.08	.17
Grouping and comparing	1.07	0.44	0.11	.00	1.52	−0.07	0.10	.50	1.07	0.40	0.11	.00	1.52	−0.06	0.11	.57
Spatial	1.52	−0.11	0.12	.39	1.75	−0.17	0.12	.17	1.52	−0.09	0.13	.48	1.75	−0.24	0.14	.08
Behavior																
NL arithmetic	–	–	–	–	1.06	0.18	0.07	.01	–	–	–	–	1.06	0.20	0.07	.00
Mental/Body arithmetic	1.00	0.13	0.21	.57	1.17	−0.02	0.10	.81	1.00	0.17	0.23	.49	1.17	0.11	0.05	.03
Instructional gesture	1.65	−0.18	0.13	.18	1.14	0.11	0.09	.26	1.65	−0.15	0.14	.27	1.14	0.07	0.10	.46
Questioning	1.45	0.50	0.14	.00	1.45	0.65	0.16	.00	1.45	0.46	0.14	.00	1.45	0.65	0.16	.00
Planning	1.11	0.22	0.10	.04	1.39	−0.11	0.15	.48	1.11	0.21	0.11	.06	1.39	−0.01	0.11	.90

Note. Values reflect numbers of utterances or actions observed rescaled, such that 1 = 0 utterances, 2 = 1 to 5 utterances, 3 = 6 to 10 utterances, 4 = 11 to 15 utterances, and 5 = 16 or more utterances. NL, number line. Covariates in preregistered model are student grade level. Teacher observations for arithmetic on the number line were not recorded due to an observation protocol error in the Pilot Study that was fixed in the Efficacy Study. Total observations: $N = 96$; Pilot Study observations $n = 43$; Efficacy Study observations $n = 53$. The control means from the bivariate models (first from left) represent the regression model intercept. The control means are used to replace the control means for the preregistered multiple linear regression models because the control means in these models were lower than 1 due to the inclusion of grade level covariates.

Will playing Fraction Ball affect students and teachers' STEM-related behaviors?

The treatment impacts on student engagement with number line arithmetic (βs = 0.30 to 0.79, ps < .001–.01, ≈ 24–63 more words per class period) and mental/body arithmetic (βs = 0.11 to 0.32, ps < .001–.01, ≈ 8–25 more words per class period) were consistent and robust to model specifications. Impacts were larger on conducting arithmetic on the number line than on conducting arithmetic off the number line, such as by using one's body or mental visualizations (e.g., finger counting, air drawing). There were mixed impacts on students' instructional gesturing in the Pilot Experiment (βs = −0.21 to −0.25, ps = .059–.095, ≈ 16–20 more words per class period); students used significantly less instructional gesturing than in PE, which was robust to model specification. However, in the Efficacy Experiment, the direction of the impacts was positive (βs = 0.19 to 0.20, ps < .001–.01, ≈ 15–16 more words per class period) and consistent across model specifications.

Teachers' engagement with number line arithmetic was only measured in the Efficacy Experiment (due to a printing error) and shows a consistent positive impact of the Fraction Ball games (βs = 0.18 to 0.20, ps < .001–.01, ≈ 14–16 more words per class period). Teachers' questioning increased across experiments and was consistent across regression specifications (βs = 0.46 to 0.65, ps < .001, ≈ 36–52 more words per class period). There were no other significant impacts of the Fraction Ball intervention on teachers' use of mental/body arithmetic, instructional gesture, and planning that were robust to experimental and regression specification.

Is engagement on the Fraction Ball scoreboard (number line) associated with greater production of rational number arithmetic language in students and teachers?

To determine whether students used greater fraction or decimal language when they engaged in number line arithmetic, we compared the frequency of rational number language used when the students were observed engaged in arithmetic on the number line with when they were not engaged in arithmetic on the number line. Our sample for this analysis contains only students participating in the treatment condition because the counterfactual condition was regular PE activities on a field not containing a number line. We conducted a paired-samples t test to contrast rational number language within student groups when engaged in arithmetic on the number line with when engaged in arithmetic not on the number line (e.g., arithmetic conversation happening in other areas of the court). Across both experiments, students produced significantly more fraction words, $t(25) = 3.47$, $p = .002$, and decimal words, $t(25) = 2.15$, $p = .042$, when they were engaging in arithmetic on the number line than when they were doing other Fraction-Ball-related activities. Groups' means and standard deviations are shown in Table 6.

Discussion

We examined the effects of playing Fraction Ball games on student and teacher production of mathematical language and behavior. Compared with a PE class, students and teachers who played Fraction Ball produced significantly more fraction and decimal language. These findings were consistent across model-specific sensitivity checks, indicating that Fraction Ball games make a large and statistically significant contribution on the production of rational number language in a typical PE period. There were marked increases in student engagement with number line arithmetic as well as significant increases in teachers' use of questioning during the Fraction Ball games.

Students and teachers participating in Fraction Ball games used rational number language 1 to 5 times every 3 min in a 50-min class period, meaning that fraction and decimal words were uttered about 16 to 83 more times by both teachers and students in a single class period. The amount of students' engagement on the number line and teachers' use of questioning occurred 16 to 83 times within each class session. In contrast, students and teachers who played Fraction Ball did not consistently increase their whole number of utterances, mental/body arithmetic, counting, patterns, gestures, spatial talk, questioning, and planning compared with students and coaches who engaged in PE activities. The PE activities we observed were already rich in whole number language and behaviors such as the

Table 6
Association between students' use of fraction and decimal words when engaging in number line arithmetic

Dependent variable	Engaging in number line arithmetic				Not engaging in number line arithmetic				t	p
	n	M	SD	CI	n	M	SD	CI		
Fractions	25	2.56	0.58	[2.32, 2.79]	25	2.05	.57	[1.82, 2.29]	3.47	.002
Decimals	25	1.77	0.38	[1.61, 1.93]	25	1.53	.46	[1.34, 1.72]	2.15	.040

Note. n represents individual 3-min cycles observed. All data are from the treatment group because the control group did not use a number line in the counterfactual condition. Two-tailed paired t test p values are reported. *CI*, confidence interval.

use of gestures, noticing patterns, questioning, grouping and comparing, and spatial talk. Therefore, we might not have found significant increases in the amount of mental/body arithmetic because the game scripts focused more on encouraging students to make connections between the representations of the number line and rational number arithmetic. This is an important step in developing rational number knowledge (Fuchs et al., 2013; Siegler & Lortie-Forgues, 2014).

Previous studies with younger populations suggest that language may bootstrap reasoning about mathematical concepts by offloading cognitive resources (Vukovic & Lesaux, 2013), and emerging work with older children implies that this mechanism may be particularly important for more complex mathematics (Peng et al., 2020). Rational numbers represent an intricate concept for most fifth and sixth graders (Booth & Newton, 2012); thus, it is likely that children would see greater benefits of language when learning about fractions and decimals. This idea is supported by the significant impact of Fraction Ball on student and teacher rational number language and student rational number understanding as in Bustamante and colleagues (2022). This study is a first step toward understanding the causal role of mathematical language with more intricate mathematical concepts.

Fraction Ball advances the PLL initiative as the first PLL installation that focuses exclusively on a specific academic domain within mathematics (i.e., fractions/decimals) and is in alignment with multiple state standards around rational numbers. This narrower focus and specificity may have implications for the impacts on student learning. The core structure of the Fraction Ball games and court design were drawn directly from research in the science of learning, and we built on and refined these ideas by co-designing the basketball court and games with teachers to incorporate teachers' expertise around the rational number curriculum and students' everyday challenges around rational number concepts. Fraction Ball is the first teacher-designed and -guided PLL where children are expected to follow a coordinated set of rules with integral roles for each player. The increased student and teacher use of fraction/decimal vocabulary and number line arithmetic suggests success in employing principles from the science of learning to inform game design. Designing the scoreboard to reflect decimals and fractions side by side may have influenced discussions about the equivalence of magnitudes represented as fractions and decimals. These findings suggest that Fraction Ball promotes the type of language and interactions that are predictive of later math outcomes and may also be causally related to improvements in math knowledge.

Our school partner teachers identified how English language learners could benefit from this exposure to standard-based math vocabulary in addition to experiences with measurement, such as on a number line. Solving problems involving measurement and conversions is a fourth-grade Common Core standard that many students are not able to cover for various reasons. As they move into fifth and sixth grades, they must play catch-up on this knowledge as a foundation to learn new and increasingly complex Common Core standards. A strategic and timely introduction of Fraction Ball and the standard-based math vocabulary it elicits could provide additional opportunities for students to strengthen understanding of mathematical concepts and vocabulary.

Teacher guidance afforded the opportunity for teachers to engage in teaching moments at the end of each game through prompts that were included in their scripts as well as additional opportunities that teachers saw. In one of our co-design sessions, the teachers emphasized the importance of mathematical accuracy in the representation of 0.66 repeating on the basketball court—not a detail of the design that we were initially attuned to. This partnership with the teachers ensured that the experience was aligned with their existing classroom practices to support integration into the daily math language practices and support teacher agency. Our joint work revealed ongoing innovative instructional approaches that the math and PE teachers had been working on. For example, the fourth author and the PE coach have been developing a series of activities that would naturally incorporate math vocabulary into everyday PE, such as sectioning the field into quadrants and referencing the quadrants using graphing vocabulary or saying "run the perimeter" when students are asked to run around the field. In addition, the PE teacher committed to making Fraction Ball one of his PE units, aligning it with the math curriculum scope and sequence, providing cross-departmental exposure to both the math skills and language that Fraction Ball elicits.

Limitations and future directions

Recent evidence from an evaluation study of Fraction Ball suggested that children in the intervention also improved in their math knowledge compared with children who participated in PE activities (Bustamante et al., 2022), providing complementary evidence that rational number language is tightly related to learning rational number concepts. Yet, the complexity of the active ingredients in this intervention precludes a more formal causal analysis of the role of math language in the learning gains reported in these experiments. The pattern of language impacts in the current study, the greater use of fraction and decimal words, align with the math learning impacts in the efficacy study, with the largest impacts being on students' performance on fraction and decimal conversions. The observational methods used in our study, inability to blind study procedures/participants, and inability to record video and audio made it impractical to track each student's language/behavior production during game play on the court to match it to the student's score on the rational number assessment.

The Fraction Ball games were compared with regular PE class because they were designed as a supplement, but Fraction Ball as described in the scripts in the supplementary material was not empirically supported, nor was it designed as a substitute for regular math or PE instruction. The control group engaged in a variety of activities during the PE class, such as stretching on the turf, running drills, and playing kickball, soccer, and other PE games. This poses an important limitation in our study because different comparison groups may allow for a richer understanding of the impacts of the manipulated design features. For example, future work might compare the language used during Fraction Ball language with the language used during a regular basketball game or during an in-class activity engaging the same fraction and decimal concepts. Such a comparison could demonstrate how the design features of Fraction Ball differ from any other basketball-related activity where shooting and scoring elicit math vocabulary.[1]

The involvement of teachers in the co-design process of the Fraction Ball games may limit the generalizability of our findings. The advantage of involving the same teachers in the analysis is that there is increased alignment between the games and teachers' instructional practices. On the other hand, teachers' involvement may have influenced changes in their regular behavior and treatment of students, potentially inflating the amount of math language used during naturalistic observations. In future work, we plan to address this limitation by co-designing with teachers who are not concurrently implementing the games.

Unfortunately, the naturalistic observations employed in the current study did not capture the extent of student and teacher talk. Rather, they captured the frequency of students' and teachers' math vocabulary use at the word level. Due to this limitation, future studies should measure the types of conversations taking place to understand the processes underlying the observed language impacts and knowledge formation—especially to determine how debriefing activities that took place on the number line after each round of the game can pedagogically enhance math talk and deepen student understanding. After each round, the teacher called the whole class over to the number line and had the students point out who was ahead, by how much, and which shots the other team would need to take to catch up. Qualitative interaction analysis of students' and teachers' discussions and problem solving during this activity could reveal the process through which students improve their rational

[1] We thank an anonymous reviewer for making this recommendation.

number reasoning from peer and teacher feedback. Furthermore, by pairing qualitative and quantitative analyses, the relation between math talk and students' conceptual and procedural understanding can be better understood and explicitly designed for (see Note 1).

Our coding protocol did not distinguish math-related gestures, patterns, spatial talk, questioning, and grouping and comparing specific to rational numbers; thus, the nature of each of the behaviors coded within these categories may be vastly different when observed in Fraction Ball versus PE activities. One possible explanation for why many of the behaviors were not produced more often during Fraction Ball may be that codes were at too rough of a grain size to capture important qualitative differences between groups' behaviors. For example, we were able to capture students' instances of counting fractions, but we did not capture peer support or corrections during that counting process. More refined coding protocols could focus on the key language mechanisms that support students' rational number arithmetic to better understand the impacts of this intervention on the learning process. Additional modifications and adaptations to the Fraction Ball games could provide novel future directions to co-design with teachers and students and test how to optimally design games to facilitate students' rational number reasoning. One potential idea is to build on the current basketball game structure of 2-point and 3-point shots to teach fractions of units that go beyond 1, such as one-third, two-thirds, and three-thirds of 2 points and one-third, two-thirds, and three-thirds of 3 points—especially when working with older students who may seek more challenging shots beyond the 3-point line (see Note 1).

Conclusion

Fraction Ball games elicited greater fraction and decimal number language use in teachers and students during playful and socially engaging activities compared with the control group. This study suggests that the language environment created by Fraction Ball may support students' rational number reasoning. These results amplify the scope of Playful Learning Landscapes to create a bridge between guided play activities that foster children's cognitive development.

Author contributions

All authors contributed to the design and implementation of the study as well as analyses and preparation of the manuscript.

Data availability

Data will be made available on request.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jecp.2023.105777.

References

Alvarez-Vargas, D., Lopez Perez, J. P., Bermudez, V., Beltrán-Grimm, S., Santana, E., Begolli, K., & Bustamante, A. S. (2023). Evidence-based designs for physically active and playful math learning. *Theory Into Practice, 62*(2), 166–180. https://doi.org/10.1080/00405841.2023.2202131.

Barsalou, L. W. (2008). Grounded cognition. *Annual Review of Psychology, 59*(1), 617–645. https://doi.org/10.1146/annurev.psych.59.103006.093639.

Begolli, K. N., & Richland, L. E. (2016). Teaching mathematics by comparison: Analog visibility as a double-edged sword. *Journal of Educational Psychology, 108*(2), 194–213. https://doi.org/10.1037/edu0000056.

Berkowitz, T., Schaeffer, M. W., Maloney, E. A., Peterson, L., Gregor, C., Levine, S. C., & Beilock, S. L. (2015). Math at home adds up to achievement in school. *Science, 350*(6257), 196–198. https://doi.org/10.1126/science.aac7427.

Booth, J. L., McGinn, K. M., Barbieri, C., Begolli, K. N., Chang, B., Miller-Cotto, D., Young, L. K., & Davenport, J. L. (2017). Evidence for cognitive science principles that impact learning in mathematics. In D. C. Geary, D. B. Berch, R. Ochsendorf, & K. M. Koepke (Eds.), *Acquisition of complex arithmetic skills and higher-order mathematics concepts* (pp. 297–325). Elsevier.

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology, 37*(4), 247–253. https://doi.org/10.1016/j.cedpsych.2012.07.001.

Bustamante, A. S., Begolli, K. N., Alvarez-Vargas, D., Bailey, D. H., & Richland, L. E. (2022). Fraction Ball: Playful and physically active fraction and decimal learning. *Journal of Educational Psychology, 114*(6), 1307–1320. https://doi.org/10.1037/edu0000714.

Bustamante, A. S., Schlesinger, M., Begolli, K. N., Golinkoff, R. M., Shahidi, N., Zonji, S., Riesen, C., Evans, N., & Hirsh-Pasek, K. (2020). More than just a game: Transforming social interaction and STEM play with Parkopolis. *Developmental Psychology, 56*(6), 1041–1056. https://doi.org/10.1037/dev0000923.

Cannon, J., Levine, S., & Huttenlocher, J. (2007). *A system for analyzing children and caregivers' language about space in structured and unstructured contexts*. Spatial Intelligence and Learning Center: Technical report. Retrieved from http://www.silccenter.org/media/silc_pdfs/resources/testsandinstruments/Spatial%20Language%20Coding%20Manual%201-10-12.pdf.

Carpenter, T. P., Fennema, E., & Romberg, T. A. (2012). *Rational numbers*. An integration of research. Routledge.

Chow, J. C., & Jacobs, M. (2016). The role of language in fraction performance: A synthesis of literature. *Learning and Individual Differences, 47*, 252–257. https://doi.org/10.1016/j.lindif.2015.12.017.

Dackermann, T., Fischer, U., Nuerk, H.-C., Cress, U., & Moeller, K. (2017). Applying embodied cognition: From useful interventions and their theoretical underpinnings to practical applications. *Zdm, 49*(4), 545–557. https://doi.org/10.1007/s11858-017-0850-z.

DeWolf, M., Bassok, M., & Holyoak, K. J. (2015). From rational numbers to algebra: Separable contributions of decimal magnitude and relational understanding of fractions. *Journal of Experimental Child Psychology, 133*, 72–84. https://doi.org/10.1016/j.jecp.2015.01.013.

Durkin, K., & Rittle-Johnson, B. (2012). The effectiveness of using incorrect examples to support learning about decimal magnitude. *Learning and Instruction, 22*(3), 206–214. https://doi.org/10.1016/j.learninstruc.2011.11.001.

Eason, S. H., & Ramani, G. B. (2018). Parent–child math talk about fractions during formal learning and guided play activities. *Child Development, 91*(2), 546–562. https://doi.org/10.1111/cdev.13199.

Fisher, K. R., Hirsh-Pasek, K., Newcombe, N., & Golinkoff, R. M. (2013). Taking shape: Supporting preschoolers' acquisition of geometric knowledge through guided play. *Child Development, 84*(6), 1872–1878. https://doi.org/10.1111/cdev.12091.

Fuchs, L. S., Schumacher, R. F., Long, J., Namkung, J., Hamlett, C. L., Cirino, P. T., Jordan, N. C., Siegler, R., Gersten, R., & Changas, P. (2013). Improving at-risk learners' understanding of fractions. *Journal of Educational Psychology, 105*(3), 683–700. https://doi.org/10.1037/a0032446.

Gaudreau, C., Bustamante, A. S., Hirsh-Pasek, K., & Golinkoff, R. M. (2021). Questions in a life-sized board game: Comparing caregivers' and children's question-asking across STEM museum exhibits. *Mind, Brain, and Education, 15*(2), 199–210. https://doi.org/10.1111/mbe.12283.

Gordon, R., & Ramani, G. B. (2021). Integrating embodied cognition and information processing: A combined model of the role of gesture in children's mathematical environments. *Frontiers in Psychology, 12*, 650286. https://doi.org/10.3389/fpsyg.2021.650286.

Gunderson, E. A., & Levine, S. C. (2011). Some types of parent number talk count more than others: Relations between parents' input and children's cardinal-number knowledge. *Developmental Science, 14*(5), 1021–1032. https://doi.org/10.1111/j.1467-7687.2011.01050.x.

Gunderson, E. A., Spaepen, E., & Levine, S. C. (2015). Approximate number word knowledge before the cardinal principle. *Journal of Experimental Child Psychology, 130*, 35–55. https://doi.org/10.1016/j.jecp.2014.09.008.

Hassinger-Das, B., Bustamante, A., Hirsh-Pasek, K., & Golinkoff, R. M. (2018). Learning Landscapes: Playing the way to learning and engagement in public spaces. *For Peer Review, 8*, 1–46. https://doi.org/10.3390/educsci8020074.

Hassinger-Das, B., Jordan, N. C., & Dyson, N. (2015). Reading stories to learn math: Mathematics vocabulary instruction for children with early numeracy difficulties. *Elementary School Journal, 116*(2), 242–264. https://doi.org/10.1086/683986.

Hassinger-Das, B., Palti, I., Golinkoff, R. M., & Hirsh-Pasek, K. (2020). Urban Thinkscape: Infusing public spaces with STEM conversation and interaction opportunities. *Journal of Cognition and Development, 21*(1), 125–147. https://doi.org/10.1080/15248372.2019.1673753.

Hassinger-Das, B., Zosh, J. M., Hansen, N., Talarowski, M., Zmich, K., Golinkoff, R. M., & Hirsh-Pasek, K. (2020). Play-and-learn spaces: Leveraging library spaces to promote caregiver and child interaction. *Library & Information Science Research, 42*(1). 101002. https://doi.org/10.1016/j.lisr.2020.101002.

Hiebert, J., & Wearne, D. (1985). A model of students' decimal computation procedures. *Cognition and Instruction, 2*(3–4), 175–205. https://doi.org/10.1207/s1532690xci0203&4_1.

Hirsh-Pasek, K., Zosh, J. M., Golinkoff, R. M., Gray, J. H., Robb, M. B., & Kaufman, J. (2015). Putting education in "educational" apps: lessons from the science of learning. Psychological science in the public interest : a journal of the American Psychological Society, 16(1), 3–34. https://doi.org/10.1177/1529100615569721.

Jennings, C. M., Jennings, J. E., Richey, J., & Dixon-Krauss, L. (1992). Increasing interest and achievement in mathematics through children's literature. *Early Childhood Research Quarterly, 7*(2), 263–276. https://doi.org/10.1016/0885-2006(92)90008-M.

Jordan, K. E., Suanda, S. H., & Brannon, E. M. (2008). Intersensory redundancy accelerates preverbal numerical competence. *Cognition, 108*(1), 210–221. https://doi.org/10.1016/j.cognition.2007.12.001.

Jordan, N. C., Hansen, N., Fuchs, L. S., Siegler, R. S., Gersten, R., & Micklos, D. (2013). Developmental predictors of fraction concepts and procedures. *Journal of Experimental Child Psychology, 116*(1), 45–58. https://doi.org/10.1016/j.jecp.2013.02.001.

Kalchman, M., Moss, J., & Case, R. (2001). Psychological models for the development of mathematical understanding: Rational numbers and functions. In S. M. Carver & D. Klahr (Eds.), *Cognition and instruction: Twenty-five years of progress* (pp. 1–38). Lawrence Erlbaum.

King, Y. A., & Purpura, D. J. (2021). Direct numeracy activities and early math skills: Math language as a mediator. *Early Childhood Research Quarterly, 54*, 252–259. https://doi.org/10.1016/j.ecresq.2020.09.012.

Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk". *Developmental Psychology, 42*(1), 59–69. https://doi.org/10.1037/0012-1649.42.1.59.

Kober, N., & Rentner, D. S. (2012). *Year Two of implementing the Common Core state standards: States' progress and challenges*. Center on Education Policy.

LeFevre, J., Fast, L., Skwarchuk, S., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development, 81*(6), 1753–1767. https://doi.org/10.1111/j.1467-8624.2010.01508.x.

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology, 46*(5), 1309–1319. https://doi.org/10.1037/a0019671.

Lewis, M. R., Matthews, P. G., & Hubbard, E. M. (2015). Neurocognitive architectures and the nonsymbolic foundations of fractions understanding. In D. B. Berch, D. C. Geary, & K. M. Koepke (Eds.), *Development of mathematical cognition: Neural substrates and genetic influences* (pp. 141–160). Academic Press.

Link, T., Moeller, K., Huber, S., Fischer, U., & Nuerk, H. (2013). Trends in neuroscience and education walk the number line—An embodied training of numerical concepts. *Trends in Neuroscience and Education, 2*(2), 74–84. https://doi.org/10.1016/j.tine.2013.06.005.

Lortie-Forgues, H., Tian, J., & Siegler, R. S. (2015). Why is learning fraction and decimal arithmetic so difficult? *Developmental Review, 38*, 201–221. https://doi.org/10.1016/j.dr.2015.07.008.

Maloney, E. A., & Beilock, S. L. (2012). Math anxiety: Who has it, why it develops, and how to guard against it. *Trends in Cognitive Sciences, 16*(8), 404–406. https://doi.org/10.1016/j.tics.2012.06.008.

Moses, R., & Cobb, C. E. (2002). *Radical equations: Civil rights from Mississippi to the Algebra Project*. Beacon.

Moss, J., & Case, R. (1999). Developing children's understanding of the rational numbers: A new model and an experimental curriculum. *Journal for Research in Mathematics Education, 30*(2), 122–147. https://doi.org/10.2307/749607.

Nasir, N. S., & Hand, V. (2008). From the court to the classroom: Opportunities for engagement, learning, and identity in basketball and classroom mathematics. *Journal of the Learning Sciences, 17*(2), 143–179. https://doi.org/10.1080/10508400801986108.

Nathan, M. J. (2021). *Foundations of embodied learning: A paradigm for education*. Routledge.

Peng, P., & Lin, X. (2019). The relation between mathematics vocabulary and mathematics performance among fourth graders. *Learning and Individual Differences, 69*, 11–21. https://doi.org/10.1016/j.lindif.2018.11.006.

Peng, P., Lin, X., Ünal, Z. E., Lee, K., Namkung, J., Chow, J., & Sales, A. (2020). Examining the mutual relations between language and mathematics: A meta-analysis. *Psychological Bulletin, 146*(7), 595–634. https://doi.org/10.1037/bul0000231.

Powell, S. R., & Driver, M. K. (2015). The influence of mathematics vocabulary instruction embedded within addition tutoring for first-grade students with mathematics difficulty. *Learning Disability Quarterly, 38*(4), 221–233. https://doi.org/10.1177/0731948714564574.

Pruden, S. M., & Levine, S. C. (2017). Parents' spatial language mediates a sex difference in preschoolers' spatial-language use. *Psychological Science, 28*(11), 1583–1596. https://doi.org/10.1177/0956797617711968.

Purpura, D. J., King, Y. A., Rolan, E., Hornburg, C. B., Schmitt, S. A., Hart, S. A., & Ganley, C. M. (2020). Examining the factor structure of the home mathematics environment to delineate its role in predicting preschool numeracy, mathematical language, and spatial skills. *Frontiers in Psychology, 11*, 1925. https://doi.org/10.3389/fpsyg.2020.01925.

Purpura, D. J., & Logan, J. A. R. (2015). The nonlinear relations of the approximate number system and mathematical language to early mathematics development. *Developmental Psychology, 51*, 1717–1724. https://doi.org/10.1037/dev0000055.

Purpura, D. J., Napoli, A. R., & King, Y. (2019). Development of mathematical language in preschool and its role in learning numeracy skills. In D. C. Geary, D. B. Berch, & K. M. Koepke (Eds.), *Cognitive foundations for improving mathematical learning* (pp. 175–193). Elsevier.

Purpura, D. J., Napoli, A. R., Wehrspann, E. A., & Gold, Z. S. (2017). Causal connections between mathematical language and mathematical knowledge: A dialogic reading intervention. *Journal of Research on Educational Effectiveness, 10*(1), 116–137. https://doi.org/10.1080/19345747.2016.1204639.

Ramani, G. B., Daubert, E. N., Lin, G. C., Kamarsu, S., Wodzinski, A., & Jaeggi, S. M. (2019). Racing dragons and remembering aliens: Benefits of playing number and working memory games on kindergartners' numerical knowledge. *Developmental Science, 23*(4). e12908. https://doi.org/10.1111/desc.12908.

Ramani, G. B., Daubert, E. N., & Scalise, N. R. (2019). Role of play and games in building children's foundational numerical knowledge. In D. C. Geary, D. B. Berch, & K. M. Koepke (Eds.), *Cognitive foundations for improving mathematical learning* (pp. 69–90). Academic Press.
Ramani, G. B., Rowe, M. L., Eason, S. H., & Leech, K. A. (2015). Math talk during informal learning activities in Head Start families. *Cognitive Development, 35*, 15–33. https://doi.org/10.1016/j.cogdev.2014.11.002.
Ramani, G. B., & Scalise, N. R. (2020). It's more than just fun and games: Play-based mathematics activities for Head Start families. *Early Childhood Research Quarterly, 50*, 78–89. https://doi.org/10.1016/j.ecresq.2018.07.011.
Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development, 79*(2), 375–394. https://doi.org/10.1111/j.1467-8624.2007.01131.x.
Richland, L. E., & Begolli, K. N. (2016). Analogy and higher order thinking: Learning mathematics as an example. *Policy Insights from the Behavioral and Brain Sciences, 3*(2), 160–168. https://doi.org/10.1177/2372732216629795.
Richland, L. E., Begolli, K. N., Simms, N., Frausel, R. R., & Lyons, E. M. (2017). Supporting mathematical discussions: The roles of comparison and cognitive load. *Educational Psychology Review, 29*(1), 41–53. https://doi.org/10.1007/s10648-016-9382-2.
Richland, L. E., & Simms, N. (2015). Analogy, higher order thinking, and education. *Wiley Interdisciplinary Reviews: Cognitive Science, 6*(2), 177–192. https://doi.org/10.1002/wcs.1336.
Ridge, K. E., Weisberg, D. S., Ilgaz, H., Hirsh-Pasek, K. A., & Golinkoff, R. M. (2015). Supermarkets speak: Increasing talk among low-socioeconomic status families. *Mind, Brain, and Education, 9*(3), 127–135. https://doi.org/10.1111/mbe.12081.
Rittle-Johnson, B., Star, J. R., & Durkin, K. (2020). How can cognitive-science research help improve education? The case of comparing multiple strategies to improve mathematics learning and teaching. *Current Directions in Psychological Science, 29*(6), 599–609. https://doi.org/10.1177/0963721420969365.
Schlesinger, M., Hassinger-Das, B., Zosh, J. M., Sawyer, J., Evans, N., & Hirsh-Pasek, K. (2020). Cognitive behavioral science behind the value of play: Leveraging everyday experiences to promote play, learning, and positive interactions. *Journal of Infant, Child, and Adolescent Psychotherapy, 19*(2), 202–216. https://doi.org/10.1080/15289168.2020.1755084.
Siegler, R. S., & Lortie-Forgues, H. (2014). An integrative theory of numerical development. *Child Development Perspectives, 8*(3), 144–150. https://doi.org/10.1111/cdep.12077.
Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games—but not circular ones—improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology, 101*(3), 545–560. https://doi.org/10.1037/a0014239.
Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology, 62*, 273–296. https://doi.org/10.1016/j.cogpsych.2011.03.001.
Susperreguy, M. I., & Davis-Kean, P. E. (2016). Maternal math talk in the home and math skills in preschool children. *Early Education and Development, 27*(6), 841–857. https://doi.org/10.1080/10409289.2016.1148480.
Toll, S. W. M., & Van Luit, J. E. H. (2014a). The developmental relationship between language and low early numeracy skills throughout kindergarten. *Exceptional Children, 81*, 64–78. https://doi.org/10.1177/0014402914532233.
Toll, S. W. M., & Van Luit, J. E. H. (2014b). Explaining numeracy development in weak performing kindergartners. *Journal of Experimental Child Psychology, 124*, 97–111. https://doi.org/10.1016/j.jecp.2014.02.001.
Vamvakoussi, X., & Vosniadou, S. (2010). Aspects of secondary school students' understanding of rational numbers and their notation. *Cognition and Instruction, 28*(2), 181–209. https://doi.org/10.1080/07370001003676603.
Vukovic, R. K., & Lesaux, N. K. (2013). The language of mathematics: Investigating the ways language counts for children's mathematical development. *Journal of Experimental Child Psychology, 115*(2), 227–244. https://doi.org/10.1016/j.jecp.2013.02.002.
Wang, Y., & Siegler, R. S. (2013). Representations of and translation between common fractions and decimal fractions. *Chinese Science Bulletin, 58*(36), 4630–4640. https://doi.org/10.1007/s11434-013-6035-4.
Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review, 9*(4), 625–636. https://doi.org/10.3758/bf03196322.
Yu, Y., Shafto, P., Bonawitz, E., Yang, S. C. H., Golinkoff, R. M., Corriveau, K. H., Hirsh-Pasek, K., & Xu, F. (2018). The theoretical and methodological opportunities afforded by guided play with young children. *Frontiers in Psychology, 9*, 1–8. https://doi.org/10.3389/fpsyg.2018.01152.
Zhang, X., Hu, B. Y., Zou, X., & Ren, L. (2020). Parent–child number application activities predict children's math trajectories from preschool to primary school. *Journal of Educational Psychology, 112*(8), 1521–1531. https://doi.org/10.1037/edu0000457.
Zosh, J. M., Hirsh-Pasek, K., Hopkins, E. J., Jensen, H., Liu, C., Neale, D., Solis, S. L., & Whitebread, D. (2018). Accessing the inaccessible: Redefining play as a spectrum. *Frontiers in Psychology, 9*. https://doi.org/10.3389/fpsyg.2018.01124 1124.