Journal of Cognition and Development

ISSN: (Print) (Online) Journal homepage: www.tandfonline.com/journals/hjcd20

# Individual Differences in Attention to Analogical Relations

**Hongyang Zhao, Lindsey E. Richland, Elayne Vollman, Bella S. Lerner, Natalie Au. Yeung & Joseph Wong**

**To cite this article:** Hongyang Zhao, Lindsey E. Richland, Elayne Vollman, Bella S. Lerner, Natalie Au. Yeung & Joseph Wong (20 Dec 2024): Individual Differences in Attention to Analogical Relations, Journal of Cognition and Development, DOI: 10.1080/15248372.2024.2441684

**To link to this article:** https://doi.org/10.1080/15248372.2024.2441684

© 2024 The Author(s). Published with license by Taylor & Francis Group, LLC.

View supplementary material

Published online: 20 Dec 2024.

Submit your article to this journal

View related articles

View Crossmark data

Full Terms & Conditions of access and use can be found at
https://www.tandfonline.com/action/journalInformation?journalCode=hjcd20

JOURNAL OF COGNITION AND DEVELOPMENT
https://doi.org/10.1080/15248372.2024.2441684

Routledge
Taylor & Francis Group

OPEN ACCESS Check for updates

# Individual Differences in Attention to Analogical Relations

Hongyang Zhao[a,b], Lindsey E. Richland[a], Elayne Vollman[c], Bella S. Lerner[d], Natalie Au. Yeung[e], and Joseph Wong[f]

[a]University of California, Irvine; [b]University of Illinois, Urbana-Champaign; [c]Lake Forest College; [d]University of Miami; [e]Adobe; [f]Workiva

**ABSTRACT**

Children reasoning about the world must attend to not only visible objects but also the relations between them. For example, in mathematics classroom, it is crucial to notice not only objects in word problems, but also how they relate to each other mathematically. Attention to relations has generally been considered a function of domain knowledge or task-specific context, though we posit that individual differences in relational attention can be identified and may affect reasoning and learning across tasks. Individual differences in spontaneous relational attention were measured in a sample of 218 fifth/sixth grade children from the U.S. The latent class analysis revealed that children could be systematically grouped into four clusters based on how likely they were to attend to relational correspondences, and importantly, these differences predicted their learning from a videotaped mathematics lesson. Children who preferentially attended to relations systematically learned more from the same lesson than those who preferentially attended to objects, controlling for prior math knowledge and Executive Functions (EFs). At the same time, the latter group showed greater learning when the lesson explicitly highlighted relational correspondences, suggesting that relational attention is a key mechanism for learning, and also that this could ensure equity across students in learning from high-quality lessons.

## Introduction

Deep learning in many domains requires *attending to* and *recognizing* analogical relations between systems that on the surface may appear different; mathematics learning is a clear instance of that property (e.g., Begolli & Richland, 2016; DeWolf et al., 2016; Richland & McDonough, 2010). As noted by Rutherford and Ahlgren (1991), "mathematics explores the possible relationships among abstractions" (p. 14). Moreover, acquiring mathematics relies on relational thinking in many mathematical contexts, for example, understanding inverse relations (e.g., division is the inverse of multiplication), and discerning the commonalities and differences among fractions, decimals, and percentages (Morsanyi, 2020).

Everyday classroom mathematics instruction similarly relies upon students noticing commonalities across representations and generating key abstractions, such as noticing

**CONTACT** Hongyang Zhao zhaoh23@uci.edu School of Education, University of California, 2036 Education, Irvine, CA 92697, USA

Supplemental data for this article can be accessed online at https://doi.org/10.1080/15248372.2024.2441684

© 2024 The Author(s). Published with license by Taylor & Francis Group, LLC.
This is an Open Access article distributed under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives License (http://creativecommons.org/licenses/by-nc-nd/4.0/), which permits non-commercial re-use, distribution, and reproduction in any medium, provided the original work is properly cited, and is not altered, transformed, or built upon in any way. The terms on which this article has been published allow the posting of the Accepted Manuscript in a repository by the author(s) or with their consent.

relations between problems (using one instructed problem to solve a new problem), between prior mathematics knowledge and new instruction, between real-world experiences and classroom experiences, or between students' solution strategies when they discuss publicly how they solved a problem (Richland & Simms, 2015).

Other domains share this phenomenon, with science as a domain relying on students recognizing relations between diagrams and real-world phenomena, or using analogies to comprehend phenomena that are too small or large to experience and understand in a concrete way (Hansen & Richland, 2020). Despite this crucial role of relational thinking in learning, much research suggests humans do not automatically notice relevant relations when available (e.g., Gick & Holyoak, 1980, 1983) and in fact often fail to generalize or transfer even when it is possible (Richland et al., 2004). Teachers vary culturally and systematically in how systematically they use verbal cues, gestures, or spatial cues to make very explicit what links they intend their students to notice (Richland, 2015; Richland et al., 2007). Therefore, variations between children in how likely they are to spontaneously notice and reason on the basis of relational correspondences in the environment may have serious consequences for learning.

Individual differences in children's spontaneous attention toward noticing relational correspondences have not been previously explored, since most research in this area focuses on children's *capacity* to reason relationally or analogically when told to do so (e.g., Gentner, 1988; Gentner & Rattermann, 1991). A growing body of work, however, has shown that doing one task that requires challenging relational thinking can prime a reasoner's attention to shift toward being more relational on a new task (Simms & Richland, 2019; Vendetti et al., 2014; Walker et al., 2018); thus, suggesting that the spontaneous tendency to attend to relations may be malleable and an important construct for understanding how children learn and derive information from novel information contexts.

The current study focuses specifically on variations among individuals in their *spontaneous* (i.e., unguided) attention to analogical relations, when there are also other sources of information and similarity available. Considerable variation in spontaneous attention to analogical relations was identified in both children and adults in unconstrained versions of relational reasoning measures used in the priming studies noted above (Simms & Richland, 2019; Vendetti et al., 2014; Walker et al., 2018). Variation was evident even among adults and children who had sufficient domain-specific knowledge to comprehend the target relations, and (for the adults), had mature executive functions (EFs), which are two factors considered crucial for individuals to *be able to* reason relationally (Goswami, 1992, 2001; Morrison et al., 2004; Rattermann & Gentner, 1998; Viskontas et al., 2004). In other words, sufficient domain knowledge and EFs alone do not guarantee that individuals will spontaneously engage in relational reasoning when explicit instruction is unavailable. This suggests that the ability to reason relationally and the likelihood of spontaneously noticing relations are distinct processes. For brevity, we use the label, "relational attention," in the remaining text to refer to attending to analogical relations in a spontaneous manner when other information is represented together with relational information and no explicit cuing to relations is available. We use the term, "relational reasoning," to refer to the capacity to reason relationally when explicit directions of such are given.

To develop a robust theory of individual differences in relational attention, two sources of information are needed. First, we need a clearer picture of how individuals vary in their relational attention patterns. Second, we need empirical evidence to support the idea that relational attention is a meaningful source of individual differences that could set itself apart from other cognitive processes closely related to relational reasoning, such as EFs. We must test whether individual differences in spontaneously noticing relevant relations are indeed consequential and may affect how new information is processed. In this current study, we test therefore whether relational attention patterns predict differences in learning from a mathematics lesson. In the following sections, we describe how the current study contributes to answering these critical questions.

### *Spontaneous relational attention patterns*

A rich literature on analogical reasoning has documented two modes of similarity-based reasoning. One is more prevalent in younger children or in those with low knowledge of a domain (Gentner, 1988; Gentner & Rattermann, 1991; Gentner & Toupin, 1986; Rattermann & Gentner, 1998), such that a reasoner is more likely to attend to similarities between featural properties of objects (e.g., color) rather than relational similarities. (e.g., the item above another). Older children and those who have accumulated relevant knowledge of a domain are typically more capable of focusing on relational similarities during their reasoning process, ignoring featural similarities which may be irrelevant or misleading, for example, noticing a correspondence between someone reaching out for something in two different scenes even if the objects were different (see Figure 1(i–ii)). It is highly plausible that individuals could show these distinct patterns in their spontaneous attention to analogical relations.

Traditionally, to study individual differences in this topic, researchers would administer an unconstrained relational mapping task and report the percentage of selecting a relational match from participants as an indicator of the extent to which individuals spontaneously attend to relation-based information, or an object match based on appearance, to represent the degree of spontaneously attending to object-based information. The drawback of this traditional method is that these separate frequency data can only reflect one dimension at a time (either relation-based or object-based information), and other possible dimensions or the *relative* frequencies between possible attentional patterns would be unrecognized. Such a methodological limitation constrains researchers from drawing conclusions about the distinct attentional patterns that individuals could have that take into account *all* the possible responses from items in the mapping task. For example, individuals could have the same, relatively low level of attending to relational similarity (e.g., Participants A and B both selected the relational match in 1 out of the 6 items), but differ in the tendency to attend to featural similarity (e.g., Participant A consistently selected the option based on featural similarity for all the other five items, whereas Participant B selected a mixture of options in these five items), thus having different attentional patterns.

Fortunately, recent statistical advancements in finite mixture modeling (McLachlan & Basford, 1988) have allowed us to investigate this question by calculating the probability of categorizing individuals based on multiple, latent dimensions within one analysis, thus providing a rigorous tool for researchers to understand the number and nature of other potential dimensions of individual differences in relational attention. With the aid of this statistical model, we would be able to identify and describe how individuals differ in their spontaneous tendency

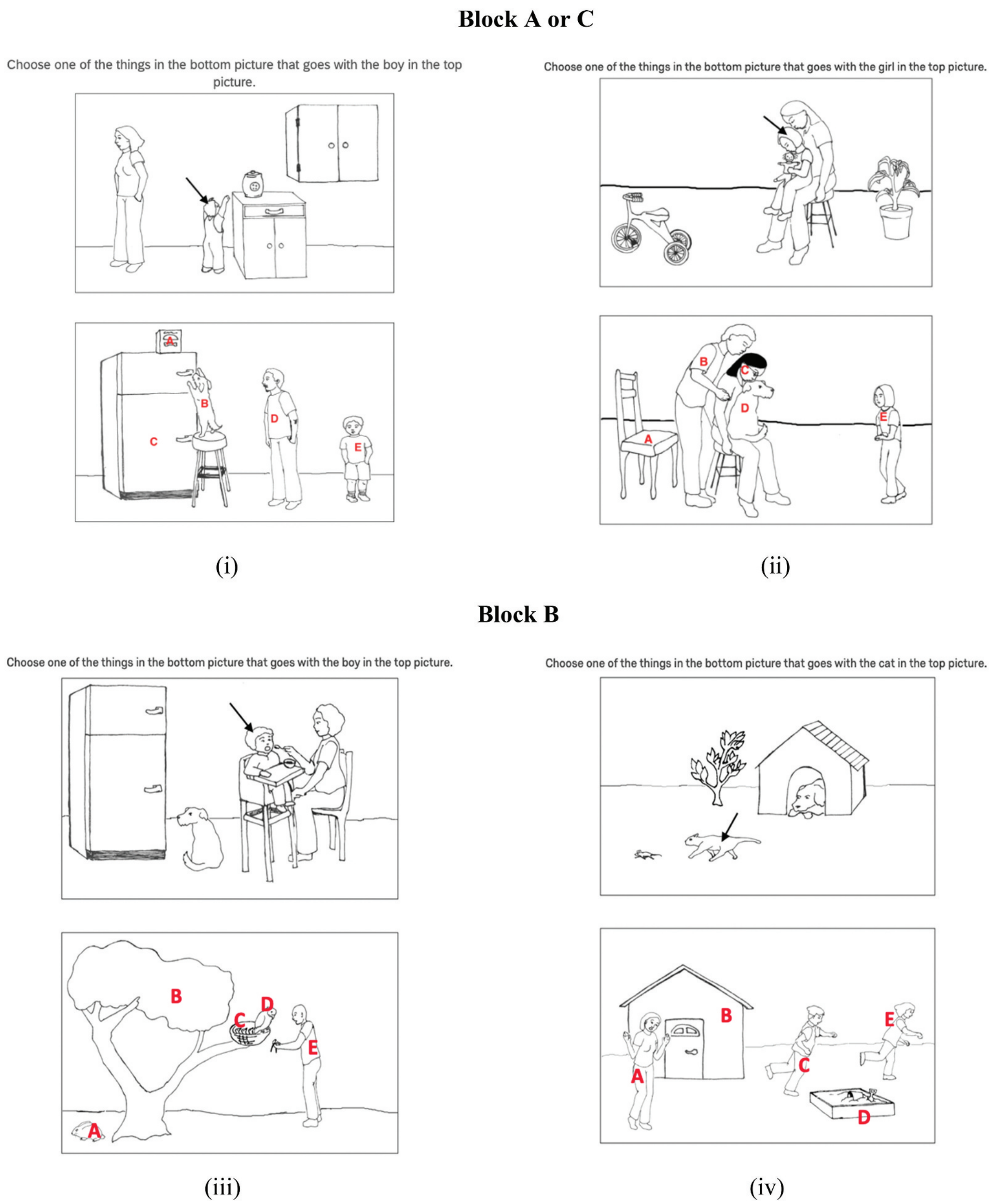


**Figure 1.** Sample items in the spontaneous scene analogy task.Note. (i) One relation with Object Match option; (ii) Two relations with Object Match option; (iii) & (iv) One relation without Object Match option.

to notice analogical relations in the presence of other distracting information. Specifically, are there certain groups of individuals that spontaneously attend to different types of information when relational and featural correspondence as well as irrelevant information are all available?

### *Contributions of relational attention to math learning*

Another essential piece of information that warrants future research is whether spontaneous relational attention is a meaningful source of individual differences or, whether it is merely a manifestation or artifact of the shared underpinning cognitive processes, such as Executive Functions, which are known to be correlated contributors to analogical reasoning (e.g., Begolli et al., 2018; Morrison et al., 2011; Richland & Burchinal, 2013). Further, we aim to understand whether this is a meaningful distinction by examining if relational attention has an identifiable effect on how new stimuli is processed. In the current study, we test whether relational attention, as measured on one task, predicts learning from a different relational reasoning opportunity – a mathematics instructional analogy. If relational attention is a meaningful construct and functions as predicted, individuals with different attentional patterns should benefit differently from the same instructional analogy, as well as demonstrate different learning patterns when explicit directions to relations, or relational language, is present versus not. We sought to demonstrate that individuals could still vary in their relational attention even if they possess the same level of EFs, and that this unique individual difference matters in their math learning.

### *Relational process and inhibitory control*

Being able to reason relationally requires individuals to inhibit the natural tendency to attend to salient, yet non-relational information, and control their limited cognitive resources to select and focus on the relational one that may not appear obvious (for discussion, see Richland et al., 2006; Richland & Zhao, 2023). Inhibitory control (IC), the ability to control attention to inhibit interfering information or previously activated cognitive processes (Diamond, 2013), can be considered one of the core components of EFs (Miyake et al., 2000). Findings from various behavioral, computational, and neurophysiological studies suggested that IC was critical for relational reasoning ability (Doumas et al., 2018; Krawczyk et al., 2008; Morrison et al., 2004, 2011; Richland & Burchinal, 2013; Thibaut et al., 2010a, 2010b). For example, Richland & Burchinal (2013) analyzed a longitudinal behavioral dataset and found that comprehensive EF and IC skills in early elementary school predicted relational reasoning ability at age 15 years old, after controlling for a number of contributing factors. Krawczyk et al. (2008) and Morrison et al. (2004) studied patients with frontotemporal lobar degeneration (FTLD), and found those with a damage in the prefrontal cortex, the brain area that is mainly responsible for EFs, performed worse than those with damage in the temporal lobe on relational reasoning tasks when the task requires higher level of inhibition of distracting information. Taken together, such evidence suggests that IC is closely related to the ability to reason relationally.

Due to the close connection between relational reasoning and relational attention, it is highly likely that IC also plays a role in the likelihood that individuals are able to attend to analogical relations over other distracting information. At the same time, it is not clear how much of the variability in children's spontaneous tendency to attend to analogical relations is related to their *ability* to disattend to salient object-level correspondences in favor of relations, versus how much of the variability can be attributed to children's *likelihood* of noticing and aiming to attend to relations. If IC completely explains the variance, it suggests that relational attention may not be a meaningful source of individual difference and that most children will attend to relations when they are able to control and direct their attention – meaning that measures of relational attention are capturing IC individual differences. If not, however, we may explain the additional variance through children's intended focus on relations versus their lack of such variation. Although we believe that various processes, including both cognitive and social processes, are involved in shaping the relational attention and that spontaneous relational attention should not be completely determined by IC, it is important to empirically test this alternative hypothesis.

### *Relational process and math learning*

Analogy-based math instructions, referring to the instructions that include direct comparisons between problems or solutions for analogical reasoning, were found to improve procedural skill, conceptual knowledge, and flexibility (Begolli & Richland, 2016; Richland & McDonough, 2010; Rittle-Johnson & Star, 2007; Star & Rittle-Johnson, 2009). During such instructions, teachers compare different solutions to the same problem, problems that target the same concept but with different surface-level features, or related concepts that share some common process. They provide ample opportunities for students to attend to analogical relations, thus making it a perfect instructional style to test the unique contributions of relational attention to math learning beyond the known contributors, such as prior knowledge and EFs (Abreu-Mendoza et al., 2020; Begolli et al., 2018).

Although analogs and comparisons are included in all analogy-based math instructions, the instructions still vary in the level of highlighting the similarities and differences among the source and target problems/solutions/concepts, which has consequences for learning (Richland & McDonough, 2010; Richland et al., 2006). For example, teachers may vary in their use of comparative gestures (e.g., gesture back and forth between problems for comparison) and relational language (e.g., "Let's think about the similarities between these solutions."), as well as whether the source and target are visually aligned and both visible throughout the instruction (Richland & McDonough, 2010). The difference in these instructional strategies results in different levels of support for students to attend to and identify analogical relations. When teachers provide more support for relational processes through these instructional strategies (i.e., a relation-heightened analogy instruction), there is a more explicit cuing that directs students' attention to the analogical relations, and vice versa (i.e., a traditional analogy instruction without the aforementioned support for attending to analogical relations). Although researchers have identified overall positive effects of high-level instructional support for relational process in the aggregate, it remains unclear how the difference in those instructional strategies interacts with individual differences in noticing analogical relations in a spontaneous manner. Do children who spontaneously attend to analogical relations when no or low-level instructional support is present, benefit more from such instruction because they can take advantage of the affordances of the intervention and gain an even deeper conceptual understanding? Or, do children who spontaneously prioritize appearance over relations, benefit more from an instruction that highlights relations better because they might not notice the key higher-order relations otherwise? The investigation of the question will deepen our understanding of the relation between instructional strategies and relational processes.

The current study aims to fill these gaps by answering the following research questions:

(1) What latent profiles of spontaneous attention to analogical relations will emerge for children in fifth and sixth grades?

(2) To what extent will relational attention patterns predict math-learning outcomes after participating in a math lesson on proportional reasoning, when prior knowledge and inhibitory control are controlled for?
(3) Will there be an interaction effect between children's relational attention pattern and type of intervention (i.e., relation-heightened analogy instruction vs. traditional analogy instruction) in predicting math learning outcomes after controlling for prior math knowledge and inhibitory control?

## Methods

### Participants

Participants in the current study were 272 children (104 girls; 87 did not report their sex) aged 10–13 (*Mean* = 11.47 yrs; *SD* = .66) in Grades 5 and 6 from five local schools (ten classrooms) in a suburban area of a large city on the west coast of US. The self-identified demographics of the participants were as follows: 25.4% White ($n = 69$), 14.7% Asian ($n = 40$), 13.6% as Latinx ($n = 37$), 1.1% Black or African American ($n = 3$), 0.7% Native Hawaiian or Pacific Islander ($n = 2$), 2.6% mixed ($n = 7$), 0.4% Native American or Alaska Native ($n = 1$), 4.0% other ($n = 11$), and 5.9% preferred not to disclose ($n = 16$). 86 participants did not report their race (31.6%). 20.2% of the participants ($n = 55$) reported English was not their primary language.

### Overall design and procedure

The current study consisted of 2 days with 1 day in between. The study was conducted online due to the COVID-19 pandemic causing local schools to switch to remote learning, so students participated remotely as part of their math class instruction. Day 1 began with a pretest math question and then participants were randomly assigned to one of the two versions of an interactive video-based math lesson. Following this, students completed the Spontaneous Scene Analogy task and then took an immediate posttest assessment to measure learning from the math lesson. Two days later, students were provided links to a delayed posttest assessment, a basic demographic survey, and an IC task (i.e., Flanker). While students were instructed to complete both parts of the study on the assigned day with 1 day in between, given the online/at-home format of the study, we gave students access to each part for a few extra days to allow more time for students to participate.

### Spontaneous scene analogy task

The Spontaneous Scene Analogy Task (see Murphy et al., 2021 for test details) was adapted from the Scene Analogy Task (Richland et al., 2006). The task contained eight multiple-choice questions with five options each. For each item, participants were presented with two pictures depicting relation (s) that were the same in both scenes but using different objects (Figure 1) (e.g., "boy reaching for cookies" and "dog reaching for bones." Participants were instructed to "choose one of the things in the bottom picture that goes with the xxx (e.g., 'boy') in the top picture." For each item, there was always a relational-match option in the target scene which corresponded to the object to be mapped in the source scene based on its relation to other object(s). For example, the boy reaching for cookies was a relational match with the dog reaching in the bottom picture (Figure 1(i)). The items varied in (a) the number of relevant relations that needed to be mapped (one or two); (b) the presence of an object in the target scene that was either featurally similar (object match) or dissimilar to the object to be mapped in the source scene.

The one-relation and two-relation items differed in that the inactive object in the one-relation items (i.e., the jar/box reached by the boy/dog in Figure 1(i)) played an active role in the two-relation items (i.e., the girl/woman being kissed was also kissing a doll/dog in Figure 1(ii)). The object-match option was an object in the target scene (e.g., the boy in Figure 1(i)) that looked similar to the object to be mapped in the source scene but did not play the same role in relation to other objects, hence making it an object match rather than a relational one. Under the current directions, if a participant picked this option, it provided an instance of evidence that they prioritized featural similarity over relational similarity.

In addition to these two options, there were also options corresponding to relational errors and irrelevant objects. Relational error refers to the object that comprised part of the relational structure but did not play the same role as the source object highlighted by the arrow in the top scene. For example, the box of treats that the dog reached out for (option A) in the *Reaching* item was considered a relational error since it was not the agent who initiated the action just like the source object, but rather an object that was impacted by the reaching action, which corresponded to the jar on the table in the top scene. Similarly, options B and D were considered relational errors in the *Kissing* item because although they were involved in the kissing action, neither the adult nor the dog were both kissing and being kissed at the same time, which was the role that the source object played in the top scene. An irrelevant object refers to the object(s) that did not play any role in the relation, such as the refrigerator (option C) and the adult (option D) in the *Reaching* item, and the chair (option A) in the *Kissing* item.

The spatial location, type of object, and animacy were all varied systematically, and the number of objects was always the same across scenes. This measure aimed to capture participants' spontaneous tendency to prioritize certain types of information when relational and object-related information is both present and no explicit cuing to either one is available. Therefore, there is no such "accuracy" score as in more traditional cognitive or educational performance measures.

The eight items were divided into three blocks on Qualtrics. Blocks A and C each included two two-relation items and one one-relation item. All of them had an object-match option. The order of the two blocks was counterbalanced. Block B was different and included two one-relation items *without an object-match option*. The three blocks were set in this way because Block A measured children's spontaneous attention, and Block B served as a training block in which the reasoner did not see an object match, thus a reasoner has the opportunity to attend to cues in their environment. This would train the reasoner that object matches could not be a consistent way to solve these challenges. The third Block C allowed us to assess the extent to which participants were able to recognize from the context and potentially switch their attention to relations when completing the third block if they had not done so in the first block.

### *Inhibitory control task*

The flanker task is an executive functioning test of inhibitory control (Eriksen & Eriksen, 1974). With the study taking place remotely, this task was administered via Tatool Web, an online, open-source software (Von Bastian et al., 2013) for running online and offline computer-based experiments that has been used by many researchers in the field of cognitive science/cognitive psychology. According to previous studies that used the Tatool Web to administer the flanker task, the reliability index ranged from 0.78 to 0.93 (e.g., Hartanto et al., 2023; Hartanto & Yang, 2019; Rey-Mermet et al., 2019), suggesting acceptable reliability. The module was embedded as the final section of

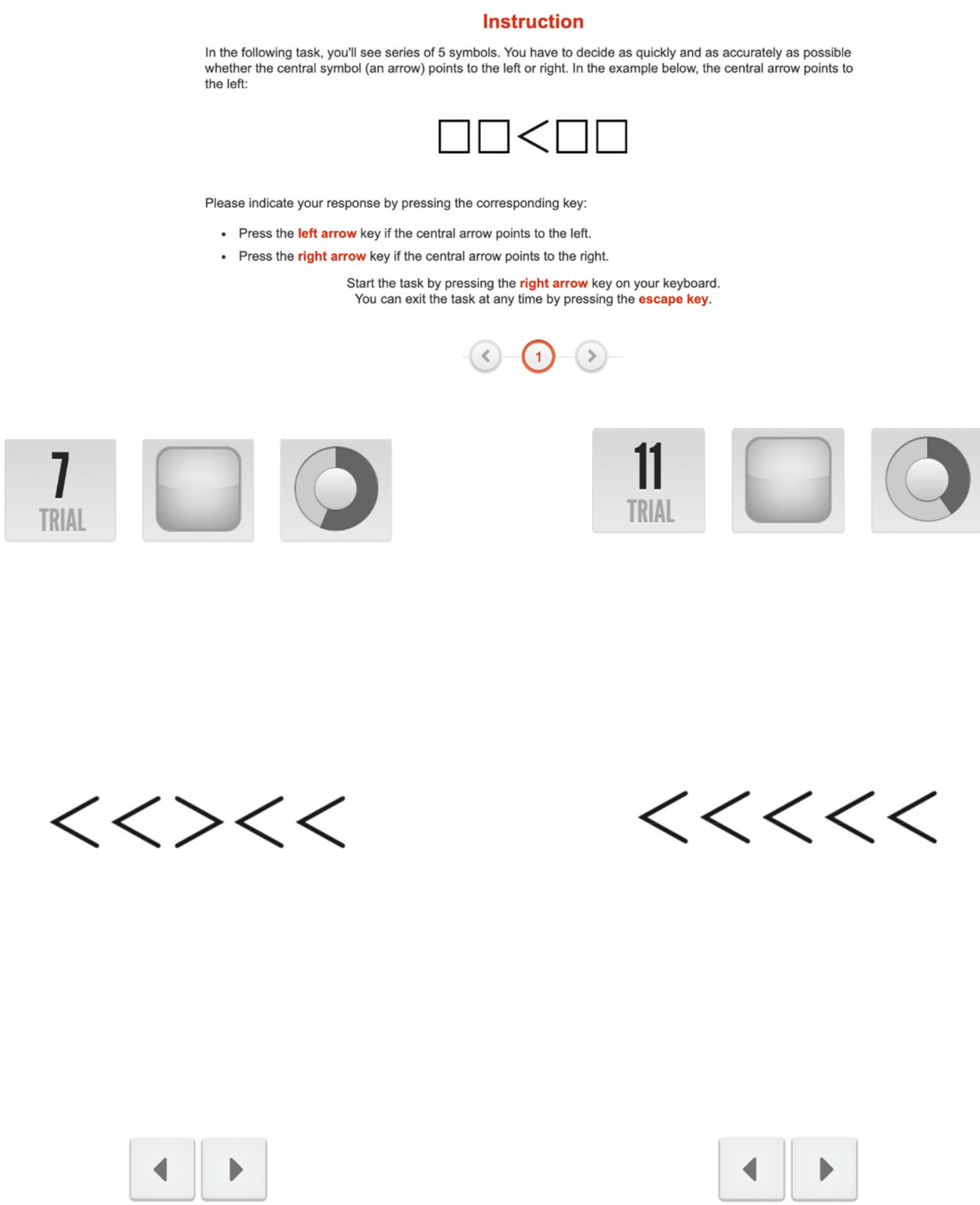


**Figure 2.** Sample items in the executive functioning (Flanker) task.

the Qualtrics study on Day 2. Participants were presented with five symbols, and were instructed to press the arrow key associated with the central symbol (i.e., an arrow that points left or right) as quickly and accurately as possible, while ignoring the surrounding irrelevant stimuli (Figure 2). The task consisted of five practice trials followed by 20 test trials that each allowed 2 seconds before progressing to the next trial, measuring participants' reaction time and accuracy in demonstrating selective attention to the target stimuli. The 25 trials were equally divided into three conditions (i.e., neutral, congruent, incongruent). In the neutral condition, a central arrow was surrounded by boxes that did not indicate any direction. In the congruent condition, the central arrow was surrounded by four arrows pointing in the same direction, whereas in the incongruent condition, the central arrow was surrounded by four arrows all pointing in the opposite direction. Participants' inhibitory control was represented by the difference in the mean accuracy score between the neutral and incongruent conditions (i.e., neutral mean accuracy – incongruent mean accuracy).

## Math lesson and measures of learning outcome

### Recording of math lesson

The researchers worked with a teacher and a curriculum designer to design a lesson introducing ratio and proportional knowledge. The 20-minute teacher-led lesson was recorded as a live-semi -scripted lesson teaching two proportional reasoning strategies: equivalent fraction strategy and unit ratio strategy (see Supplemental Materials: Sections A and B for more details). The main learning objective of the lesson was for students to learn the procedures of the two solution strategies to solve proportional reasoning problems, but moreover, while both strategies could be employed to solve the word problems, a specific strategy is more efficient based on the numerical properties of the problem.

To create the video lesson recording, the teacher taught a diverse class of fifth- and sixth-grade students who were recruited for the recording of the lesson. This design procedure, with real students, allows for the natural variability introduced by everyday teaching

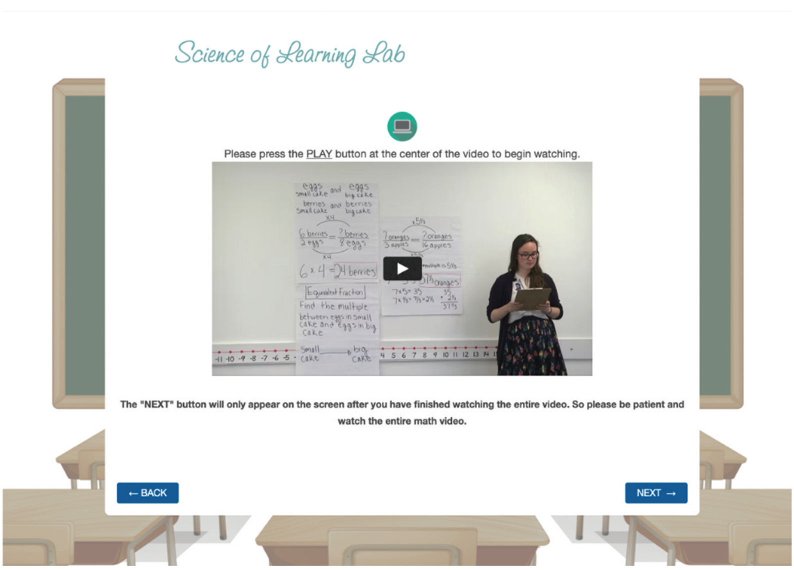


Figure 3. Screenshot of the math lesson video on Qualtrics.

contexts with students having real-world conversations regarding the solution strategies. The procedures therefore enable instructional stimuli to have high ecological validity while maintaining experimental control (Begolli & Richland, 2018). The lesson was divided into nine, two-minute video clips and they were embedded into a Qualtrics survey (Figure 3). Students could pause the video, but could not rewind or fast-forward. Between the video clips, students answered questions to promote learning: the instructor would pose a question, then students would solve the problem and provide their answer. After students completed the question, the next video clip would follow. There were no response time constraints. Please see Supplemental Materials for the details of the content and instructional design of the math lesson.

### Relation-heightened analogy instruction vs. Traditional analogy instruction

Within classrooms, students were randomly assigned to either receive a relation-heightened analogy instruction (intervention condition) that had a stronger support in directing students' attention to analogical relations or a traditional analogy instruction with less of such support (control condition). All students watched a high-quality video lesson on proportional reasoning, but the conditions differed systematically in key instructional moments in the lesson. The intervention condition included verbal and gesture instruction that highlights relations paired with relation-eliciting prompts and questions for students to engage in analogical reasoning (Richland, 2015). For example, the intervention condition included interwoven comparison across problem types and solution strategies, a focus on abstraction, and teacher-highlighted opportunities for inference (e.g., thinking about the mathematical principles of "why," "how," and "when" in problem-solving context; Kazemi & Hintz, 2014; Richland et al., 2004; Smith et al., 2009). When doing so, the teacher drew learners' attention to the key comparisons in the problem context and facilitated relational reasoning by using relational language (e.g., "Let's think about the similarities between these solutions." "So we've used the denominators to figure out the multiple between the small cake and big cake, or between, and the small juice and big juice.") and linking gestures across to highlight the comparison relationships (Alibali et al., 2014; Richland, 2015). Students also received prompts in Qualtrics to generate analogical reasoning utterances during the lesson by comparing solution strategies and problem types. For example, students completed multiple choices and filled in the blank questions to compare units in the problem [e.g., "Fill in the blank to match the parts of the two problems: 3 apples (in the second problem) is like ___ (in the first problem)"].

The control condition received the same visual information and math content during the lesson. However, the verbal instruction excluded all instances of relational language and was replaced with sequential, procedural discussion of the strategies. The questions throughout the lesson were also replaced with procedural information and recall questions (e.g., "What is the missing ingredient in the cake problem?"). Finally, there was no use of gestures that facilitated analogical reasoning in the lesson in the control condition.

### Math tests

The math pretest was administered prior to watching the video lesson on Day 1. Students solved an open-ended proportional reasoning problem on their own using any strategy they knew. The problem was as follows: "Alex is making a strawberry cake. To make a small cake, the recipe calls for 2 eggs and 6 strawberries. Alex wants to make a big cake, so he uses 8 eggs. How many strawberries will Alex need in order to make a big cake?" No feedback was provided after participants completed the item. Participants' responses were coded as "1" for correct and "0" for incorrect.

Learning gains were assessed at two time points: on an immediate posttest after receiving the math lesson on Day 1 and on a delayed posttest on Day 2. The immediate posttest included two open-ended proportional reasoning items similar to the pretest item and the problems covered in the math lesson, which asked participants to reason through the proportional relations with calculations of up to two-digit numbers, but with different cover stories and numbers, for example, "An electric bike can go 5 miles in 7 minutes. How many minutes will it take for this bike to go 10 miles?" In addition, participants were instructed to solve each problem using a specific strategy (i.e., equivalent fraction or unit ratio). The delayed posttest included six additional similar proportional items with different cover stories and numbers (i.e., Biking, Reading, Toy Car, Smoothie, Train, Truck problems). We made sure that the topics of the cover stories are familiar to children of our age group. Participants were instructed to answer the first two items with the equivalent fraction strategy, the following two with the unit ratio strategy, and the last two with the better strategy between the two. No feedback was provided for any of the problems. Participants' responses were coded as "1" and "0" for accuracy only, regardless of strategy use. Two summary scores of accuracies were calculated for the immediate and delayed posttests, respectively. The internal consistency reliability of the overall accuracy across these two assessment time points was acceptable (Cronbach's $\alpha = 0.77$).

## Results

### Data inspection and descriptive data analysis

Data inspection and treatment of missing data are explained in Supplemental Materials: Section C. All analyses were completed in the SPSS 28 (IBM Corp, 2021) and Mplus 8.0 (Muthen & Muthen, 2017) with the Full-Information Maximum Likelihood estimation.

Table 1 presents the descriptive data on the frequency and proportion of participants' performance on the Spontaneous Scene Analogy Task. In *Block A* and *C*, each picture scene had four categories as options (i.e., Irrelevant Object, Relational Match, Object Match, Relational Error; items in Block A and C), the most frequently chosen option was the Relational Match (*Mean* = 47.89%; *Range* = 42.40% to 67.89%), except for one item. It was followed by the Object Match (*Mean* = 29.43%; *Range* = 14.22% to 49.54%) and Relational Error (*Mean* = 16.28%; *Range* = 4.13% to 34.40%). For the two items in *Block B*, which had three categories for options (without Object Match), on average, 45.65% of participants chose the Relational Match with a considerable proportion of participants choosing the Irrelevant Object (*Mean* = 33.26%) or Relational Error (*Mean* = 20.87%). Therefore, our data showed that a relatively large proportion of participants spontaneously attended to similarities based on relations while completing the Spontaneous Scene Analogy Task.

For the IC task, participants completed around nine Flanker trials per condition on average (*Range* = 9.17 to 9.24). See Table 2 for the average accuracy scores and reaction times for the 171 participants who started the Flanker task.

For the math pretest, 84% of the 212 participants ($n = 184$) correctly answered the pretest question, meaning they had the relevant prior knowledge to benefit from the lesson. After

**Table 1.** Descriptive results of the spontaneous scene analogy task.

| | | | Categories | | | | |
|---|---|---|---|---|---|---|---|
| Block | Item | Number of Relations | Irrelevant Object | Relational Match | Object Match | Relational Error | Total |
| A | *Kiss* | 2 | 6 (2.75%) | 94 (43.12%) | 37 (16.97%) | 75 (34.40%) | 213 (97.71%) |
| | *Reach* | 1 | 22 (10.09%) | 148 (67.89%) | 31 (14.22%) | 11 (5.05%) | 212 (97.25%) |
| | *Crash* | 2 | 15 (6.88%) | 60 (27.52%) | 108 (49.54%) | 29 (13.30%) | 212 (97.25%) |
| B | *Chase* | 1 | 98 (44.95%) | 96 (44.04%) | – | 24 (11.01%) | 218 (100%) |
| | *Feed* | 1 | 47 (21.56%) | 103 (47.25%) | – | 67 (30.73%) | 218 (100%) |
| C | *Yell* | 2 | 5 (2.29%) | 92 (42.20%) | 54 (24.77%) | 62 (28.44%) | 213 (97.71%) |
| | *Pull* | 1 | 35 (16.06%) | 93 (42.66%) | 76 (34.86%) | 9 (4.13%) | 213 (97.71%) |
| | *Tow* | 2 | 12 (5.50%) | 95 (43.58%) | 79 (36.24%) | 27 (12.39%) | 213 (97.71%) |

**Table 2.** Descriptive results of the EF task.

| Condition | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Neutral | | | Incongruent | | | Congruent | | |
| *n* (SD) | Accuracy (SD) | Reaction Time *ms* (SD) | *n* (SD) | Accuracy (SD) | Reaction Time *ms* (SD) | *n* (SD) | Accuracy (SD) | Reaction Time *ms* (SD) |
| 9.17 (3.02) | 0.75 (0.27) | 981.11 (328.93) | 9.24 (2.98) | 0.50 (0.29) | 1159.92 (316.14) | 9.18 (3.14) | 0.83 (0.24) | 909.59 (322.35) |

The table presents the means and standard deviations (in parenthesis) for 171 participants by condition. *n* = number of completed Tatool trials. Accuracy = the proportion of Tatool trials with the correct response. Reaction Time = the time took by participants to complete the Tatool trials (in milliseconds).

the lesson, the average proportion of correct responses on the immediate posttest was 0.71 ($n = 209$; $SD = 0.34$) and on the delayed posttest was as follows: 0.78 ($n = 186$; $SD = 0.27$).

### *Identifying latent clusters of relational attention*

Our first research question was identifying the latent clusters of children's relational attention that could best account for the heterogeneity in response patterns of the full sample on the Spontaneous Scene Analogy items. Considering the two items in Block B of the Spontaneous Scene Analogy task included only three categories as options (Items *Chase* and *Feed*), whereas the other 6 items (Items *Kiss*, *Reach*, *Crash*, *Yell*, *Pull*, *Tow*) included four, the two items in Block B were not included in the latent class analysis to ensure consistent and interpretable results. The overall analyses involved three steps: 1) specifying the latent class model; 2) determining the number of latent classes; and 3) inspecting and interpreting the latent classes. Due to space limitations, technical details are presented in Section D in the Supplemental Materials, and major results are summarized here.

Table 3 shows the log-likelihood, AIC, BIC, BLRT results, and cluster sizes for the 1-cluster to 4-cluster solutions. Based on the recommended best practices (Lanza et al., 2013), we determined that the 4-cluster model had the best model fit to our sample data in that it had meaningful clusters, the lowest AIC, second lowest BIC, and the bootstrapped likelihood ratio test suggested an improved model-fit over the 3-cluster model. Thus, the 4-cluster model was elected to be the final solution.

**Table 3.** Model-fit indices for the four mixture models.

| Number of Latent Classes | Log Likelihood | AIC | BIC | Class Size Distributions | BLRT |
|---|---|---|---|---|---|
| 1 | −1420.57 | 2877.13 | 2938.05 | 100% | – |
| 2 | −1280.75 | 2635.49 | 2760.72 | 49.1%, 50.9% | $\Delta 2LL = 279.64$, $df = 17$, $p < 0.001$ |
| 3 | −1210.71 | 2533.42 | 2722.95 | 51.8%, 12.4%, 35.8% | $\Delta 2LL = 140.08$, $df = 19$, $p < 0.001$ |
| 4 | −1185.16 | 2520.31 | 2774.15 | 50.4%, 23.9%, 13.3%, 12.4% | $\Delta 2LL = 51.11$, $df = 18$, $p = .013$ |

AIC = Akaike Information Criteria; BIC = Bayesian Information Criteria.

**Table 4.** Description of the four latent classes based on the response patterns to the spontaneous scene analogy items.

| | | | Item | | | | | |
|---|---|---|---|---|---|---|---|---|
| Class | Class Size | Category | *Kiss* | *Reach* | *Crash* | *Yell* | *Pull* | *Tow* |
| Relational Attender | 110 (50.4%) | Irrelevant Object | 0.0% | 1.9% | 5.5% | 0.9% | 12.6% | 2.9% |
| | | Relational Match | 56.5% | 92.9% | 47.4% | 57.8% | 80.1% | 79.6% |
| | | Object Match | 0.0% | 1.1% | 33.8% | 4.7% | 5.0% | 9.4% |
| | | Relational Error | 43.4% | 4.1% | 13.3% | 36.6% | 2.3% | 8.1% |
| Emerging Relational Attender | 52 (23.9%) | Irrelevant Object | 3.9% | 2.3% | 6.3% | 4.4% | 10.7% | 0.0% |
| | | Relational Match | 58.1% | 83.1% | 17.7% | 46.2% | 0.0% | 13.5% |
| | | Object Match | 13.0% | 9.2% | 67.6% | 49.4% | 89.3% | 81.0% |
| | | Relational Error | 25.0% | 5.3% | 8.4% | 0.0% | 0.0% | 5.4% |
| Object Matcher | 29 (13.3%) | Irrelevant Object | 0.0% | 6.1% | 0.0% | 0.0% | 4.8% | 3.5% |
| | | Relational Match | 3.3% | 8.7% | 0.0% | 13.9% | 19.2% | 6.9% |
| | | Object Match | 89.6% | 85.2% | 90.0% | 73.6% | 76.0% | 85.2% |
| | | Relational Error | 7.1% | 0.0% | 10.0% | 12.5% | 0.0% | 4.4% |
| Inconsistent Responder | 27 (12.4%) | Irrelevant Object | 16.0% | 65.1% | 23.1% | 7.5% | 55.2% | 29.2% |
| | | Relational Match | 15.5% | 19.5% | 0.0% | 11.9% | 0.0% | 0.0% |
| | | Object Match | 15.7% | 0.0% | 47.9% | 11.9% | 20.3% | 17.5% |
| | | Relational Error | 52.8% | 15.4% | 29.0% | 68.7% | 24.4% | 53.3% |

To interpret the four latent clusters, we inspected their overall response patterns on the Spontaneous Scene Analogy task. Table 4 presents the proportion of participants' selection of each category option for each of the six items within each latent class. The first class had a high proportion of choosing the relational match option (*Range* = 47.4% to 80.1%) for all the 6 items compared to other options, suggesting these participants were likely to be "Relational Attenders," who had the tendency to consistently attend to similarities based on relations. For the second class, participants had a high proportion of choosing the relational match option only for half of the items with an equally high proportion of choosing the object match option for the other half, suggesting their attention to relational similarities emerged but was not as developed as the Relational Attenders. Therefore, they were labeled as "Emerging Relational Attenders." The third class was named to be "Object Matchers," who consistently attended to similarities based on the appearance of objects as evidenced by the fact that they had an overwhelmingly high proportion of choosing the Object Match option for the six items (*Range* = 73.6% to 90.0%). The fourth class presented a relatively mixed response pattern. For five out of the six items, the most frequent option selected by these participants was either the Irrelevant Object or Relational Error, and for the other item, the most frequent option was the Object Match. It suggested that these participants tended to have an inconsistent pattern in attending to similarities, thus were labeled as "Inconsistent Responders."[1]

We further examined whether the four latent classes of participants were related to how participants' attention shifted in Block B, where there was no longer an object match available, and in the third block (A or C), when there was again an object match. Specifically, we sought to characterize whether the classes were associated with differences in responses to the "training" items in Block B, to determine whether participants shifted away from an object matching strategy toward relational responding, based on cues from the task itself.

Specifically, we identified the two groups of participants within each class based on the presenting order of Block A and C (i.e., participants who completed Block A first vs. participants who completed Block C first). We documented the proportion of their selection of each category option for the three items within each block. If any specific class was found to have a considerable, increased selection rate (≥10%) of the Relational Match option from the first presented three items to the subsequent three items (referred to as "the relationally responsive trend" in the following text), it would suggest that the class was particularly sensitive to relational cues, thus was adept in shifting their attention to focus on similarities based on relations, if otherwise not.

Importantly, the results (Table 5) indicated that the relationally responsive trend was more consistently observed for the Relational Attenders and Emerging Relational Attenders, who showed this pattern in 3 out of the 6 items with an average increased proportion of 16.7%, than the Object Matchers and Inconsistent Responders. It suggests that Relational Attenders and Emerging Relational Attenders may be more likely to pick up the relational cues from the task practice compared to the Object Matchers and Inconsistent Responders.

### *Relating latent class of relational attention to math learning outcomes*

Our second research question examined how participants' attention to analogical relations would predict their math learning outcomes, while controlling for prior math knowledge and IC. Moreover, we sought to test the hypothesis that children with the same level of prior math knowledge and IC but with different patterns of relational attention would benefit differently from a same math lesson, as well as have different patterns in their learning outcomes between the two types of math interventions. Figure 4 shows the overall mixture model tested, and we followed practice recommendations (Asparouhov & Muthén, 2014; Lanza et al., 2013; Wang & Wang, 2019) and used the manual three-step approach to test the mixture model. Technical details are provided in Section E of the Supplemental Materials.

[1]The four classes were replicated with another sample of the similar age group (Sample 2: $n = 267$; Grade: 5 and 6). For the replication analysis, we followed the same procedures as those reported here. Specifically, a total of five mixture models, from a 1-cluster model up to 5-cluster model, were fit to the Sample 2 data on the same 6 items in the Spontaneous Scene Analogy Task. The four-cluster solution was still identified as the best solution (See Table 1 in Supplemental Materials for details on the model-fit indices). A close inspection of the response patterns for the four classes in Sample 2 revealed that they were all matched to the four classes reported here (See Table 2 in Supplemental Materials for more details). The proportion of the four clusters in Sample 2 was also found to be comparable to their counterparts reported here.

**Table 5.** Observed proportions of responses to the spontaneous scene analogy items by presenting order and by class.

a. Relational Attenders

| | | Presenting order | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Block A first (*n*=64) | | | | Block C first (*n*=45) | | | |
| Block | Item | Irrelevant Object | Relational Match | Object Match | Relational Error | Irrelevant Object | Relational Match | Object Match | Relational Error |
| A | *Kiss* | 0.0% | 57.8% | 0.0% | 42.2% | 0.0% | 54.5% | 0.0% | 45.5% |
| | *Reach* | 1.6% | 93.8% | 1.6% | 3.1% | 2.3% | 93.2% | 0.0% | 4.5% |
| | *Crash** | 7.8% | 39.1% | 35.9% | 17.2% | 2.3% | 63.6% | 27.3% | 6.8% |
| C | *Yell** | 0.0% | 64.5% | 3.2% | 32.3% | 2.2% | 48.9% | 6.7% | 42.2% |
| | *Pull** | 9.7% | 88.7% | 0.0% | 1.6% | 20.0% | 68.9% | 8.9% | 2.2% |
| | *Tow* | 3.2% | 82.3% | 3.2% | 11.3% | 2.2% | 75.6% | 17.8% | 4.4% |

b. Emerging Relational Attenders

| | | Presenting order | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Block A first (*n*=14) | | | | Block C first (*n*=38) | | | |
| Block | Item | Irrelevant Object | Relational Match | Object Match | Relational Error | Irrelevant Object | Relational Match | Object Match | Relational Error |
| A | *Kiss* | 7.1% | 64.3% | 7.1% | 21.4% | 2.9% | 52.9% | 17.6% | 26.5% |
| | *Reach** | 0.0% | 71.4% | 21.4% | 7.1% | 2.9% | 88.2% | 2.9% | 5.9% |
| | *Crash* | 7.1% | 21.4% | 71.4% | 0.0% | 5.9% | 11.8% | 70.6% | 11.8% |
| C | *Yell** | 16.7% | 58.3% | 25.0% | 0.0% | 0.0% | 39.5% | 60.5% | 0.0% |
| | *Pull* | 16.7% | 0.0% | 83.3% | 0.0% | 5.3% | 0.0% | 94.7% | 0.0% |
| | *Tow** | 0.0% | 25.0% | 75.0% | 0.0% | 0.0% | 10.5% | 81.6% | 7.9% |

c. Object Matchers

| | | Presenting order | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Block A first (*n*=18) | | | | Block C first (*n*=11) | | | |
| Block | Item | Irrelevant Object | Relational Match | Object Match | Relational Error | Irrelevant Object | Relational Match | Object Match | Relational Error |
| A | *Kiss* | 0.0% | 0.0% | 88.9% | 11.1% | 0.0% | 9.1% | 90.9% | 0.0% |
| | *Reach* | 5.6% | 0.0% | 94.4% | 0.0% | 9.1% | 9.1% | 81.8% | 0.0% |
| | *Crash* | 0.0% | 0.0% | 88.9% | 11.1% | 0.0% | 0.0% | 90.9% | 9.1% |
| C | *Yell** | 0.0% | 17.6% | 64.7% | 17.6% | 0.0% | 9.1% | 81.8% | 9.1% |
| | *Pull* | 5.9% | 23.5% | 70.6% | 0.0% | 0.0% | 18.2% | 81.8% | 0.0% |
| | *Tow** | 5.9% | 11.8% | 76.5% | 5.9% | 0.0% | 0.0% | 100.0% | 0.0% |

(*Continued*)

**Table 5.** (Continued).

d. Inconsistent Responders

| | | Presenting order | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Block A first ($n$=15) | | | | Block C first ($n$=12) | | | |
| Block | Item | Irrelevant Object | Relational Match | Object Match | Relational Error | Irrelevant Object | Relational Match | Object Match | Relational Error |
| A | *Kiss* | 13.3% | 20.0% | 13.3% | 53.3% | 16.7% | 8.3% | 16.7% | 50.0% |
| | *Reach** | 66.7% | 13.3% | 0.0% | 20.0% | 63.6% | 27.3% | 0.0% | 9.1% |
| | *Crash* | 26.7% | 0.0% | 40.0% | 33.3% | 18.2% | 0.0% | 54.5% | 27.3% |
| C | *Yell** | 0.0% | 20.0% | 20.0% | 60.0% | 16.7% | 0.0% | 0.0% | 83.3% |
| | *Pull* | 66.7% | 0.0% | 6.7% | 26.7% | 41.7% | 0.0% | 33.3% | 25.0% |
| | *Tow* | 20.0% | 0.0% | 6.7% | 73.3% | 41.7% | 0.0% | 33.3% | 25.0% |

$n$=participants within each class who completed the blocks in the specified order. *. The items in which the relationally responsive trend was observed (the average rate of choosing the Relational Match option for the three items shown in the second block increased by 10% or more compared to that for the three items shown in the first block).

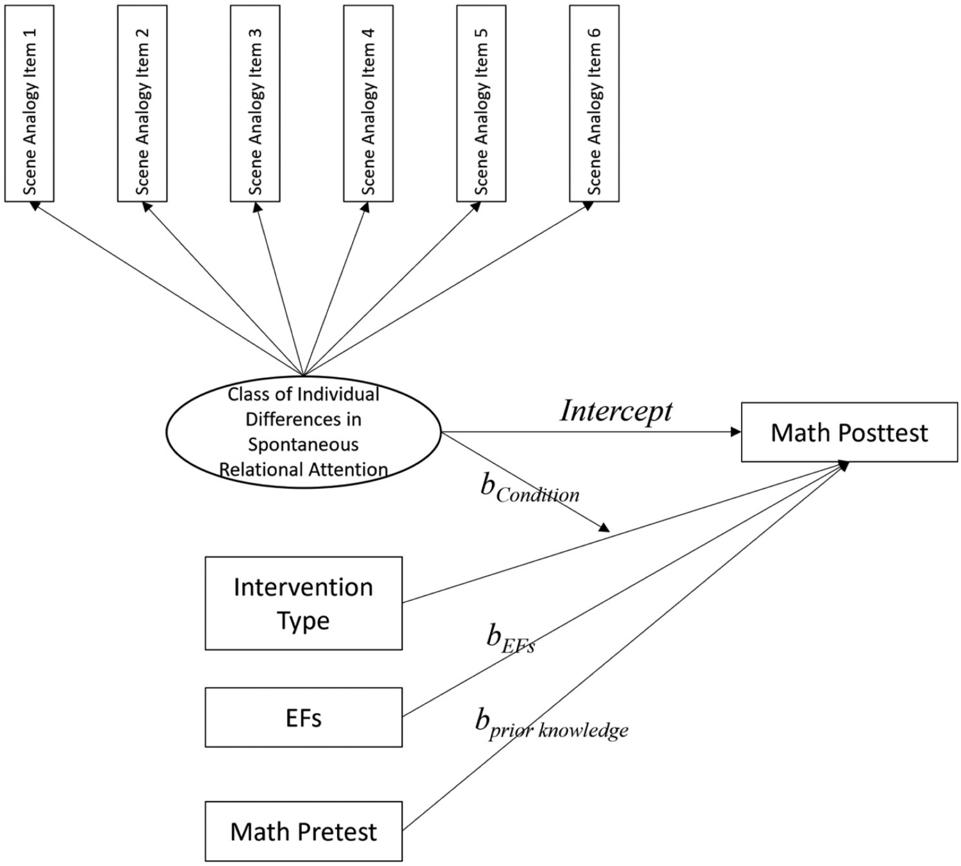


**Figure 4.** Mixture model relating latent class membership of relational attention to math learning outcomes. Note. The latent classes of individual differences in spontaneous relational attention were obtained from analyzing the participants' response patterns on the six items in the Spontaneous Scene Analogy task. The same model was specified in two separate analyses with the math immediate posttest and delayed posttest as the outcome variable, individually.

### *Overall math performance by relational attention class*

Results showed similar patterns for the two posttests, though the effects were stronger in the delayed posttest where there were a greater number of measurement items. See Table 6 for the unstandardized regression coefficients. For the immediate posttest, the omnibus test of the overall difference in mean scores across the four relational classes was not significant after controlling for pretest and inhibition control scores, as implied by the model comparison test [$\chi^2$ (3) = 7.46, $p = 0.058$]. The pairwise comparison tests suggested that after controlling for the math pretest and inhibition control scores, no significant differences were found between any pair of relational attention classes ($0.01 < |\Delta\beta_0| < 0.65$, $.056 < p < 0.978$), although the difference between the Object Matchers and Relational Attenders reflected the hypothesized trend in means ($\Delta\beta_0 = -0.60$, $p = 0.056$; See Figure 5). In the covariates, a significant main effect was

**Table 6.** Unstandardized regression coefficients of the LCA model relating the class membership of relational attention to math learning outcomes.

| Math Posttest | Class | Intercept (S.E.) | $b_{EFs}$ (S.E.) | $b_{prior\ knowledge}$ (S.E.) | $b_{Condition}$ (S.E.) |
|---|---|---|---|---|---|
| Delayed Posttest | Relational Attenders | 3.57* (0.40) | −0.42 (0.36) | 1.54* (0.42) | 0.12 (0.31) |
| | Emerging Relational Attenders | 3.76* (0.46) | −0.42 (0.36) | 1.54* (0.42) | −0.52 (0.53) |
| | Object Matchers | 1.82* (0.65) | −0.42 (0.36) | 1.54* (0.42) | 1.57* (0.65) |
| | Inconsistent Responders | 2.93* (0.69) | −0.42 (0.36) | 1.54* (0.42) | 0.46 (0.80) |
| Immediate Posttest | Relational Attenders | 1.06* (0.14) | −0.06 (0.15) | 0.61* (0.15) | −0.07 (0.12) |
| | Emerging Relational Attenders | 1.10* (0.18) | −0.06 (0.15) | 0.61* (0.15) | 0.02 (0.19) |
| | Object Matchers | 0.46 (0.34) | −0.06 (0.15) | 0.61* (0.15) | 0.47 (0.33) |
| | Inconsistent Responders | 0.45 (0.31) | −0.06 (0.15) | 0.61* (0.15) | 0.13 (0.37) |

*$p < 0.05$.

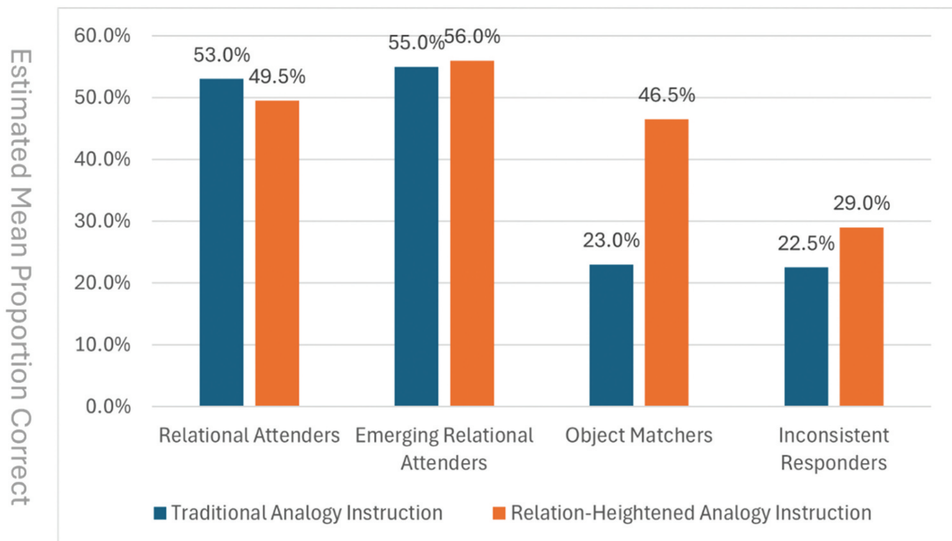


**Figure 5.** Estimated mean proportion correct of the math immediate posttest of the four latent classes.

found for prior math knowledge ($b_{Pretest} = 0.61$, $p < 0.001$), but not for inhibitory control ($b_{Inhibition\ Control} = -0.06$, $p = 0.666$) on the immediate posttest.

The same analyses were next conducted on the delayed posttest, which did not have the time constraints imposed on the immediate test, so we could include a broader range of items. After controlling for math pretest and inhibition control scores, there were significant differences in the mean score across the four relational attention classes as indicated by the Satorra-Bentler scaled chi-square test, indicating that the more complex model, assuming difference exists among classes, significantly improved the model fit compared to the model which assumed no such difference [$\chi^2$ (3) = 9.89, $p = 0.020$]. The finding supported our first hypothesis that relational attention uniquely contributes to math learning outcomes above and beyond prior knowledge and EFs. Second, pairwise comparison tests revealed that for children with the same prior math knowledge and IC, the Object Matchers were found to have significantly lower mean scores compared to the Relational Attenders ($\Delta\beta_0 = -1.75$, $S.E. = 0.54$, $p = 0.001$) and Emerging Relational Attenders ($\Delta\beta_0 = -1.95$, $S.E. = 0.56$, $p = 0.001$), whereas no significant difference was found in the other pairs ($0.20 < |\Delta\beta_0| < 0.83$, $.151 < p < 0.309$). See Figure 6 for the estimated intercepts of the four classes.

### *Math intervention condition effects*

As an omnibus test for the interaction effect between type of instruction and relational attention pattern in predicting math learning outcomes, the Satorra-Bentler scaled chi-square tests were not statistically significant for the immediate or delayed posttests [immediate: $\chi^2(3) = 2.54$, $p = 0.468$; delayed: $\chi^2(3) = 5.25$, $p = 0.155$]. See more technical details in Section E of the Supplemental Materials. Importantly, however, on the delayed posttest, a significant difference in the mean score between the two types of interventions was found in the Object Matchers, but not in the other three classes (see Table 6). On average, for children with the same level of prior math knowledge and IC, the Object Matchers who were placed in the relation-heightened intervention scored 0.48 standard deviations higher on the delayed posttest than the Object Matchers who were placed in the traditional intervention. The finding supported our hypothesis that the potential benefit that individuals gain from the relation-heightened analogy instruction compared to the traditional one depends on their spontaneous tendency to attend to relations. It's worth

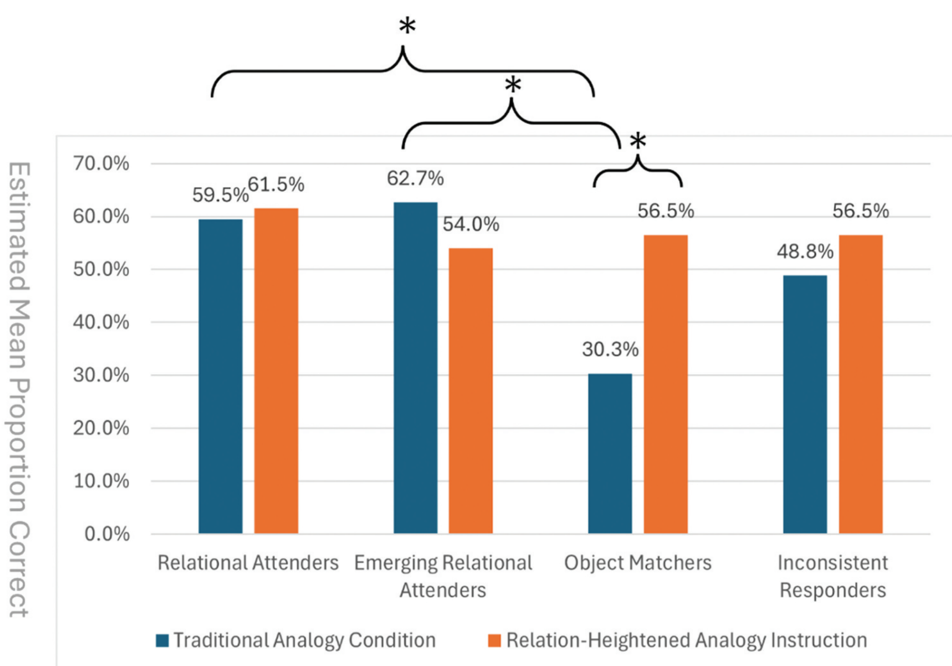


**Figure 6.** Estimated mean proportion correct of the math delayed posttest of the four latent classes. Note: *$p < 0.05$ based on pairwise comparison tests.

noting that controlling for IC and prior math knowledge, the Object Matchers, who participated in the relation-focused intervention, had a mean posttest score (i.e., 1.82 + 1.57 = 3.39) that was comparable to that of the Relational Attenders and Emerging Relational Attenders (i.e., Traditional intervention: Relational Attenders: 3.57, Emerging Relational Attenders: 3.76; Relation-heightened intervention: Relational Attenders: 3.57 + 0.12 = 3.69, Emerging Relational Attenders: 3.76–0.52 = 3.24). In addition, a significant main effect in predicting the delayed posttest score was found for prior math knowledge ($b_{Pretest} = 1.54$, $S.E. = 0.42$, $p < 0.001$), but not for IC ($b_{Inhibition\ Control} = -0.42$, $S.E. = 0.42$, $p = 0.240$).

Previous studies in the literature on expertise and pedagogical comparisons suggested a possible interaction effect between prior knowledge and math intervention condition (Kalyuga, 2007; Rittle-Johnson & Kmicikewycz, 2008; Rittle-Johnson et al., 2009), as well as between EFs and math intervention condition (Begolli et al., 2018). In other words, the learning outcomes from a math lesson that leverages pedagogical comparisons versus from one without it might be different depending on children's characteristics, including prior math knowledge and EFs. For example, Rittle-Johnson et al. (2009) found that students who have better prior knowledge benefited more from comparing different solution strategies than from studying examples sequentially or comparing problem types (instead of solutions), whereas their peers with lower prior knowledge, benefited more from the latter teaching practice. Moreover, Begolli et al. (2018) found that working memory and inhibitory control both predicted learning gains from a cognitively demanding math lesson that encouraged comparisons.

It is also likely that such interaction effects would be further complicated after considering children's spontaneous analogical attention pattern, resulting in potential three-way-interaction effects on the learning outcome from a math lesson (i.e., prior knowledge × intervention condition × spontaneous attentional pattern, EFs × intervention condition × spontaneous attentional pattern). We first examined children's mean performance on the math pretest and their EF by intervention condition and attentional pattern. See Table 3 in the Supplemental Materials. We then centered the EF accuracy data around its mean and tested another mixture model that included these three-way-interaction terms. Based on the analysis results, all our major findings from the previous analysis were held. Interestingly, we found that the relation-heightened instruction worked in different ways for Object Matchers and Inconsistent Responders. See Section E in the Supplemental Materials for more details. Since we had a small sample size of children by the time we looked at specific attentional patterns, different levels of prior knowledge and EFs, and assignment to condition (e.g., $n_{\text{inconsistent responders in the control condition}} = 11$), we were cautious in making any claim or drawing any conclusion from this more complex mixture model due to the limited power.

## Discussion and conclusions

### *Individual differences in relational attention*

In the current study, we examined individual differences in relational attention among U.S. children in fifth and sixth grades. There are several important implications. First, considerable individual differences in relational attention were found among fifth and sixth graders. Even when there was no explicit guidance to search for the relational similarities, around one-third (28%) to two-thirds (68%) of the participants spontaneously noticed the analogical relations and selected a relational match in each of our Spontaneous Scene Analogy task items. When compared across age groups, fifth and sixth graders were more likely to spontaneously notice analogical relations than children of 4-years-old (*Mean* = 0.23 to 0.35; Simms & Richland, 2019), and less likely but comparable to adults (*Mean* = 0.48 to 0.68; Vendetti et al., 2014). Moreover, when compared within the same age group, the tendency to reason based on relations when given explicit instruction to do so was much higher (i.e., 83%–97%; 13- and 14-year-olds; Richland et al., 2006). The comparison suggests that children of this age *are able to* engage in relational reasoning when direct guidance on pattern recognition is provided, but may not *spontaneously* do so. Thus, it is important for teachers and parents to recognize this tendency and then explicitly direct children's attention to analogical relations in both formal and informal learning settings when they are intending children to build on these relations for more deep learning opportunities. We will further discuss this point in the following section.

Second, four distinct latent profiles of relational attention emerged and were replicated in the current study. Based on their attentional patterns, children in Grades 5 and 6 could be categorized into Relational Attenders, Emerging Relational Attenders, Object Matchers, and Inconsistent Responders. Relational Attenders are more likely to consistently and spontaneously prioritize relations over other competing information. We describe these children as having a "relational mindset" (Vendetti et al., 2014), such that they tended to reason on the basis of the structural similarities between seemingly different systems more frequently than based on the object-level similarities. Relational Attenders were the largest cluster among the four, and together with Emerging Relational Attenders, made up close to 74% of the sample. Although a similar tendency was observed in Emerging Relational Attenders, they were not as consistent to privilege relational similarity over featural similarity as Relational Attenders, thus it is reasonable to speculate their relational attention has emerged but is still developing.

Another sample with the same age group was used to validate the latent clusters (see Footnote 1 and Supplemental Materials), and importantly we found a comparable proportional composition (i.e., Relational Attenders + Emerging Relational Attenders: 60%). These findings suggest that a significant proportion of children of this age at least start to notice analogical relations spontaneously, and are sensitive to task-related cues that indicate object features are not the most informative to this task success.

Object Matchers, on the other hand, were observed to consistently match based on featural similarity, rarely selecting to reason on the basis of analogical relations spontaneously. They seemingly tended to have difficulty in resisting the lure of noticing the surface-level similarities and focusing their attention on the less salient relational similarities, even when they had some experience with the task indicating that object similarity was not a reliable cue for detecting a problem match. While they may have selected a non-object match on Block B of the task, they went back to object-matching when that became available as an option again in Block C. Although not making up for a high proportion in our samples (13% and 16%), these children might need explicit guidance on relational reasoning most dearly to shift their spontaneous attention from featural similarity to relational similarity. Our findings also indicated that this group benefits most from an instruction that highlights comparisons and supports analogical relations, compared to their peers with other attentional patterns.

Inconsistent Responders had a less predictable attentional pattern compared to the other three groups. They were similar to Object Matchers in the low likelihood of noticing relations, but unlike Object Matchers who consistently prioritize object similarity, the Inconsistent Responders also tended to make relational errors or selected irrelevant objects. Like Object Matchers, they were not as responsive to the relational cues as Relational Attenders and Emerging Relational Attenders. One key aspect to acknowledge is that the individual differences captured in our study considered both spontaneous relational attention and relational reasoning ability, since participants needed to successfully pick a relational match option to let us know that they attended to the relational information spontaneously. Thus, it is impossible for us to entirely distinguish between these processes in our study. However, a previous study found that most children aged between 9 and 11 years demonstrated success on the version of the Scene Analogy task that assessed relational reasoning ability (Richland et al., 2006), rather than unguided attention, with a range from 93% to 87% success. Therefore, it is reasonable to speculate that for children aged between 10 and 13 in our sample, the majority of them have adequate relational reasoning skill to notice and map complex relations, if they were aware they should do so. In addition, by controlling children's prior math knowledge and EFs, which are the key contributors to relational reasoning ability, we indirectly controlled individuals' relational reasoning ability generally, if not exactly. Therefore, we believe our findings could still generally represent individual differences in their spontaneous relational attention and how they matter to learning from math classroom instructions.

Together these findings suggest that Relational Responders and Emerging Relational Responders may be likely to notice instructionally relevant relational correspondences on their own or based on cues within the instructional task, but Object Matchers and Inconsistent Responders may not. At least for Object Matchers, the instructional cues or guidance on searching for analogical relations between seemingly dissimilar systems should be as salient and straightforward as possible – a finding supported by our intervention results.

### *Relational attention in math learning*

In addition to understanding individual differences in relational attention, another goal of the current study was to test whether relational attention is a unique, meaningful construct that would contribute to everyday reasoning opportunities such as mathematics learning. Our findings support the assertion that it is a meaningful, consequential construct, and also that it cannot be completely attributed to other known individual differences such as EFs or relational knowledge.

After controlling for children's prior proportional knowledge and EFs, there were still significant differences in the math-learning outcomes among children with different attentional profiles after they participated in the same math lesson on proportional reasoning. For children with the same level of prior knowledge and EFs, those who at least start to spontaneously notice analogical relations tended to retain better learning outcomes than those who consistently focused attention on features. In our study, this pattern was only visible during the delayed math assessment, not an immediate one, likely because this measure was very short due to time-constraints. The need to conduct this study online due to the pandemic context meant we were only able to use a two-item immediate math posttest measure. Thus, this measure was likely not as sensitive as the delayed measure which was a fuller assessment. We also made the choice to include the full delayed posttest measure rather than shifting schedules to expand the same day immediate posttest because this was a more educationally valid measure of student learning, so opted to focus our analyses and interpretation on those data.

These study results have important implications, providing a new avenue to improving math outcomes through cultivating a "relational mindset" via opportunities in learning settings across development. The opportunity to train students' attention to relations could be domain-general, with training not necessarily embedded in mathematics, or in the specific mathematics being instructed, which indicates opportunities for ample intervention entry points. For example, teachers might intentionally and explicitly highlight analogical relations between seemingly different problems that share the same structure. Parents could direct children's attention to structural similarities between seemingly different objects during their daily interactions. Another finding that worth mentioning, although not the focus of this study, is that when relational attention was used to predict math learning outcomes in addition to prior math knowledge and EFs, EFs was no longer a significant contributor as previously found in other studies without a relational attention measure included (e.g., Begolli et al., 2018). It suggests that relational attention might be a more potent construct in math learning than the more basic cognition functions, such as EFs, or that EFs may function to improve mathematics outcomes by improving students' relational reasoning, which in turn improves mathematical learning.

Moreover, compared to the traditional analogy-based math instruction, the relation-heightened analogy-based math instruction particularly benefited the Object Matchers over students with other attentional patterns. This finding supports the hypothesis that students gain a deeper conceptual understanding by shifting their attention from featural to relational similarity during math instruction. The Object Matchers who received the traditional math intervention had the lowest math performance controlling for prior math knowledge and EFs, while those in the relation-focused intervention were comparable to the two higher-achieving groups of peers. Therefore, relation-heightened math instruction could be a powerful tool to enhance learning for all students. One thing to note here is that although we found the relation-heightened math intervention particularly benefited the Object Matchers, we didn't find such a benefit among other groups of children who had different attentional patterns. This suggests that there is added complexity that may be involved in the relationship between an individual's mind-set or prior knowledge, and the level of relational highlighting that might be most effective instructionally (see also Rittle-Johnson et al., 2009). Sidney and Thompson (2019), have also highlighted the importance of weighing the amount of information to be processed against the benefits of highlighting relations in an instruction.

## *Limitations*

There are several limitations that we would like to mention. First, although we found six items in the Spontaneous Scene Analogy task were enough to reliably group individuals based on their relational attention patterns, the grouping might not capture all types of relational attention. Large, community-based relational attention patterns have been posited that might mean children are socialized to attend to relations such as figure-ground and context-based reasoning (Christie et al., 2020; Masuda & Nisbett, 2001; Nisbett, 2004). Future cultural research would be useful to better clarify the nature of different forms of relational attention in reasoners' everyday lives. Second, due to time constraints, only two math items were included in our immediate posttest, which might be related to the inconsistent result patterns observed for the immediate and delayed posttests. Also due to the time constraint, only an inhibitory control measure was included in the current study as an indicator of individual difference in executive functioning. If time allows, future investigations could include more and varied EF measures and longer immediate and delayed math assessments to further investigate the interrelations among EFs, relational attention, and math learning.

In sum, we found children differ systematically in their spontaneous attention to relational similarities, and such individual differences were found to affect reasoning and learning across tasks, specifically, learning from a videotaped mathematics lesson. Children who preferentially attended to relations systematically learned more from the same lesson than those who preferentially attended to objects, controlling for prior math knowledge and Executive Functions (EFs). At the same time, the latter group showed greater learning when the lesson explicitly highlighted relational correspondences, suggesting that relational attention is a key mechanism for learning. Teachers and parents might be able to take both formal and informal learning opportunities to cultivate a relational mindset in children so as to prepare them to be alert learners in math classrooms, suggesting a novel approach to broadening students' skill in mathematics.

## Acknowledgments

The authors would like to express their sincere appreciation to Dr. Xu Li for his valuable feedback for improving the manuscript

## Disclosure statement

No potential conflict of interest was reported by the author(s).

## Funding

This work was supported by the Institute of Education Sciences (IES) under grant numbers: [R305A170488 and R305A190467]. The writing of this work was also supported by the National Science Foundation under the grant number: [2327398]. The opinions expressed are those of the authors and do not necessarily represent the views of the funders.

## ORCID

Hongyang Zhao http://orcid.org/0000-0003-4711-0314
Lindsey E. Richland http://orcid.org/0000-0003-1514-6013
Bella S. Lerner http://orcid.org/0000-0002-6957-7698
Joseph Wong http://orcid.org/0000-0003-1890-6284

## Data availability statement

The study materials, data, and analytical codes are available by contacting the first author of this study.

## Ethics approval statement

The current study was reviewed and approved by the Institutional Review Board (ID: IRB17–0683).

## Permission to reproduce materials

The authors have permission from other sources to reproduce the materials used in this study.

## References

Abreu-Mendoza, R. A., Coulanges, L., Ali, K., Powell, A. B., & Rosenberg-Lee, M. (2020). Children's discrete proportional reasoning is related to inhibitory control and enhanced by priming continuous representations. *Journal of Experimental Child Psychology*, *199*, 104931.

Alibali, M. W., Nathan, M. J., Wolfgram, M. S., Church, R. B., Jacobs, S. A., Johnson Martinez, C., & Knuth, E. J. (2014). How teachers link ideas in mathematics instruction using speech and gesture: A corpus analysis. *Cognition & Instruction*, *32*(1), 65–100. https://doi.org/10.1080/07370008.2013.858161

Asparouhov, T., & Muthén, B. (2014). Auxiliary variables in mixture modeling: Three-step approaches using Mplus. *Structural Equation Modeling: A Multidisciplinary Journal*, *21*(3), 329–341. https://doi.org/10.1080/10705511.2014.915181

Begolli, K. N., & Richland, L. E. (2016). Teaching mathematics by comparison: Analog visibility as a double-edged sword. *Journal of Educational Psychology*, *108*(2), 194. https://doi.org/10.1037/edu0000056

Begolli, K. N., & Richland, L. E. (2018). Bridging cognitive science and real classrooms: A video methodology for experimental research. *The Journal of Experimental Education*, *86*(4), 671–689. https://doi.org/10.1080/00220973.2017.1347775

Begolli, K. N., Richland, L. E., Jaeggi, S. M., Lyons, E. M., Klostermann, E. C., & Matlen, B. J. (2018). Executive function in learning mathematics by comparison: Incorporating everyday classrooms into the science of learning. *Thinking & Reasoning*, *24*(2), 280–313. https://doi.org/10.1080/13546783.2018.1429306

Christie, S., Gao, Y., & Ma, Q. (2020). Development of analogical reasoning: A novel perspective from cross-cultural studies. *Child Development Perspectives*, *14*(3), 164–170. https://doi.org/10.1111/cdep.12380

DeWolf, M., Bassok, M., & Holyoak, K. J. (2016). A set for relational reasoning: Facilitation of algebraic modeling by a fraction task. *Journal of Experimental Child Psychology*, *152*, 351–366. https://doi.org/10.1016/j.jecp.2016.06.016

Diamond, A. (2013). Executive functions. *Annual Review of Psychology*, *64*(1), 135–168.

Doumas, L. A., Morrison, R. G., & Richland, L. E. (2018). Individual differences in relational learning and analogical reasoning: A computational model of longitudinal change. *Frontiers in Psychology*, *9*, 1235.

Eriksen, B. A., & Eriksen, C. W. (1974). Effects of noise letters upon the identification of a target letter in a nonsearch task. *Perception & Psychophysics*, *16*(1), 143–149. https://doi.org/10.3758/BF03203267

Gentner, D. (1988). Analogical inference and analogical access. In A. Prieditis (Ed.), *Analogica* (pp. 63–88). Morgan Kaufmann.

Gentner, D., & Rattermann, M. J. (1991). Language and the career of similarity. In S. A. Gelman & J. P. Byrnes (Eds.), *Perspectives on thought and language: Interrelations in development* (pp. 225–277). Cambridge University Press.

Gentner, D., & Toupin, C. (1986). Systematicity and surface similarity in the development of analogy. *Cognitive Science*, *10*(3), 277–300. https://doi.org/10.1207/s15516709cog1003_2

Gick, M. L., & Holyoak, K. J. (1980). Analogical problem solving. *Cognitive Psychology*, *12*(3), 306–355. https://doi.org/10.1016/0010-0285(80)90013-4

Gick, M. L., & Holyoak, K. J. (1983). Schema induction and analogical transfer. *Cognitive Psychology*, *15*(1), 1–38. https://doi.org/10.1016/0010-0285(83)90002-6

Goswami, U. (1992). *Analogical reasoning in children*. Lawrence Erlbaum Associates.

Goswami, U. (2001). Analogical reasoning in children. In D. Gentner, K. J. Holyoak, & B. N. Kokinov (Eds.), *The analogical mind: Perspectives from cognitive science* (pp. 437–470). MIT Press.

Hansen, J., & Richland, L. E. (2020). Teaching and learning science through multiple representations: Intuitions and executive functions. *CBE - Life Sciences Education*, *19*(4), ar61.

Hartanto, A., Chua, Y. J., Quek, F. Y., Wong, J., Ooi, W. M., & Majeed, N. M. (2023). Problematic smartphone usage, objective smartphone engagement, and executive functions: A latent variable analysis. *Attention, Perception & Psychophysics*, *85*(8), 2610–2625.

Hartanto, A., & Yang, H. (2019). Does early active bilingualism enhance inhibitory control and monitoring? A propensity-matching analysis. *Journal of Experimental Psychology: Learning, Memory & Cognition*, *45*(2), 360–378.

IBM Corp. (2021). *IBM SPSS statistics for windows, version 28.0*.

Kalyuga, S. (2007). Expertise reversal effect and its implications for learner-tailored instruction. *Educational Psychology Review*, *19*(4), 509–539. https://doi.org/10.1007/s10648-007-9054-3

Kazemi, E., & Hintz, A. (2014). *Intentional talk: How to structure and lead productive mathematical discussions*. Stenhouse Publishers.

Krawczyk, D. C., Morrison, R. G., Viskontas, I., Holyoak, K. J., Chow, T. W., Mendez, M. F., & Knowlton, B. J. (2008). Distraction during relational reasoning: The role of prefrontal cortex in interference control. *Neuropsychologia*, *46*(7), 2020–2032.

Lanza, S. T., Tan, X., & Bray, B. C. (2013). Latent class analysis with distal outcomes: A flexible model-based approach. *Structural Equation Modeling: A Multidisciplinary Journal*, *20*(1), 1–26. https://doi.org/10.1080/10705511.2013.742377

Masuda, T., & Nisbett, R. E. (2001). Attending holistically versus analytically: Comparing the context sensitivity of Japanese and Americans. *Journal of Personality & Social Psychology*, *81*(5), 922–934. https://doi.org/10.1037/0022-3514.81.5.922

McLachlan, G. J., & Basford, K. E. (1988). *Mixture models: Inference and applications to clustering* (Vol. 38). M. Dekker.

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology*, *41*(1), 49–100.

Morrison, R. G., Doumas, L. A., & Richland, L. E. (2011). A computational account of children's analogical reasoning: Balancing inhibitory control in working memory and relational representation. *Developmental Science*, *14*(3), 516–529.

Morrison, R. G., Krawczyk, D. C., Holyoak, K. J., Hummel, J. E., Chow, T. W., Miller, B. L., & Knowlton, B. J. (2004). A neurocomputational model of analogical reasoning and its breakdown in frontotemporal lobar degeneration. *Journal of Cognitive Neuroscience*, *16*(2), 260–271. https://doi.org/10.1162/089892904322984553

Morsanyi, K. (2020). Reasoning skills in individuals with mathematics difficulties. In A. J. Martin, R. A. Sperling, & K. J. Newton (Eds.), *Handbook of educational psychology and students with special needs* (pp. 510–533). Routledge.

Murphy, A. N., Zheng, Y., Shivaram, A., Vollman, E., & Richland, L. E. (2021). Bias and sensitivity to task constraints in spontaneous relational attention. *Journal of Experimental Child Psychology*, *202* (104981), 1–23. https://doi.org/10.1016/j.jecp.2020.104981

Muthen, L. K., & Muthen, B. O. (2017). *Mplus user's guide* (8th ed.). Muthen & Muthen.

Nisbett, R. E. (2004). *The geography of thought: How Asians and Westerners think differently… and why*. The Free Press.
Rattermann, M. J., & Gentner, D. (1998). More evidence for a relational shift in the development of analogy: Children's performance on a causal-mapping task. *Cognitive Development*, *13*(4), 453–478. https://doi.org/10.1016/S0885-2014(98)90003-X
Rey-Mermet, A., Gade, M., Souza, A. S., von Bastian, C. C., & Oberauer, K. (2019). Is executive control related to working memory capacity and fluid intelligence?. *Journal of Experimental Psychology. General*, *148*(8), 1335–1372.
Richland, L. E. (2015). Cross-cultural differences in linking gestures during instructional analogies. *Cognition & Instruction*, *33*(4), 295–321. https://doi.org/10.1080/07370008.2015.1091459
Richland, L. E., & Burchinal, M. R. (2013). Early executive function predicts reasoning development. *Psychological Science*, *24*(1), 87–92.
Richland, L. E., Holyoak, K. J., & Stigler, J. W. (2004). Analogy generation in eighth-grade mathematics classrooms. *Cognition & Instruction*, *22*(1), 37–60. https://doi.org/10.1207/s1532690Xci2201_2
Richland, L. E., & McDonough, I. M. (2010). Learning by analogy: Discriminating between potential analogs. *Contemporary Educational Psychology*, *35*(1), 28–43. https://doi.org/10.1016/j.cedpsych.2009.09.001
Richland, L. E., Morrison, R. G., & Holyoak, K. J. (2006). Children's development of analogical reasoning: Insights from scene analogy problems. *Journal of Experimental Child Psychology*, *94*(3), 249–273. https://doi.org/10.1016/j.jecp.2006.02.002
Richland, L. E., & Simms, N. (2015). Analogy, higher order thinking, and education. *WIREs Cognitive Science*, *6*(2), 177–192. https://doi.org/10.1002/wcs.1336
Richland, L. E., & Zhao, H. (2023). Measuring the building blocks of everyday cognition: Executive functions and relational reasoning. *Frontiers in Psychology*, *14*, 1219414.
Richland, L. E., Zur, O., & Holyoak, K. J. (2007). Cognitive supports for analogies in the mathematics classroom. *Science*, *316*(5828), 1128–1129. https://doi.org/10.1126/science.1142103
Rittle-Johnson, B., & Kmicikewycz, A. O. (2008). When generating answers benefits arithmetic skill: The importance of prior knowledge. *Journal of Experimental Child Psychology*, *101*(1), 75–81. https://doi.org/10.1016/j.jecp.2008.03.001
Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology*, *99*(3), 561. https://doi.org/10.1037/0022-0663.99.3.561
Rittle-Johnson, B., Star, J. R., & Durkin, K. (2009). The importance of prior knowledge when comparing examples: Influences on conceptual and procedural knowledge of equation solving. *Journal of Educational Psychology*, *101*(4), 836. https://doi.org/10.1037/a0016026
Rutherford, F. J., & Ahlgren, A. (1991). *Science for all Americans*. Oxford University Press.
Sidney, P. G., & Thompson, C. A. (2019). Implicit analogies in learning: Supporting transfer by warming up. *Current Directions in Psychological Science*, *28*(6), 619–625. https://doi.org/10.1177/0963721419870801
Simms, N. K., & Richland, L. E. (2019). Generating relations elicits a relational mindset in children. *Cognitive Science*, *43*(10), e12795. https://doi.org/10.1111/cogs.12795
Smith, M. S., Hughes, E. K., Engle, R. A., & Stein, M. K. (2009). Orchestrating discussions. *Mathematics Teaching in the Middle School*, *14*(9), 548–556. https://doi.org/10.5951/MTMS.14.9.0548
Star, J. R., & Rittle-Johnson, B. (2009). It pays to compare: An experimental study on computational estimation. *Journal of Experimental Child Psychology*, *102*(4), 408–426. https://doi.org/10.1016/j.jecp.2008.11.004
Thibaut, J. P., French, R., & Vezneva, M. (2010). Cognitive load and semantic analogies: Searching semantic space. *Psychonomic Bulletin & Review*, *17*(4), 569–574.
Thibaut, J. P., French, R., & Vezneva, M. (2010a). The development of analogy making in children: Cognitive load and executive functions. *Journal of Experimental Child Psychology*, *106*(1), 1–19.

Vendetti, M. S., Wu, A., & Holyoak, K. J. (2014). Far-out thinking: Generating solutions to distant analogies promotes relational thinking. *Psychological Science*, *25*(4), 928–933. https://doi.org/10.1177/0956797613518079

Viskontas, I. V., Morrison, R. G., Holyoak, K. J., Hummel, J. E., & Knowlton, B. J. (2004). Relational integration, inhibition, and analogical reasoning in older adults. *Psychology and Aging*, *19*(4), 581. https://doi.org/10.1037/0882-7974.19.4.581

Von Bastian, C. C., Locher, A., & Ruflin, M. (2013). Tatool: A Java-based open-source programming framework for psychological studies. *Behavior Research Methods*, *45*(1), 108–115. https://doi.org/10.3758/s13428-012-0224-y

Walker, C. M., Hubachek, S. Q., & Vendetti, M. S. (2018). Achieving abstraction: Generating far analogies promotes relational reasoning in children. *Developmental Psychology*, *54*(10), 1833. https://doi.org/10.1037/dev0000581

Wang, J., & Wang, X. (2019). *Structural equation modeling: Applications using Mplus*. John Wiley & Sons.