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Linking Gestures: Cross-Cultural Variation During Instructional Analogies

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Deictic linking gestures, hand and arm motions that physically embody links being communicated between two or more objects in the shared communicative environment, are explored in a cross-cultural sample of mathematics instruction. Linking gestures are specifically examined here when they occur in the context of communicative analogies designed to link two distinct yet mutually informative representations. Video coding of eighth-grade mathematics lessons in the United States, Japan, and Hong Kong revealed that teachers in the higher achieving regions (Hong Kong and Japan) used reliably more linking gestures concurrent with verbal linkages than did U.S. teachers. Further, they were significantly more likely to tailor their gesture use to the recency of students' experiences with source than U.S. teachers. The overall data align with growing evidence that U.S. teachers may not systematically capitalize on pedagogical opportunities to draw linkages between representations and that gestures may play a key role in doing so.

In a coherent curriculum, mathematical ideas are linked to and build on one another so that students' understanding and knowledge deepen and their ability to apply mathematics expands (National Council of Teachers of Mathematics [NCTM] Executive Summary: Principles and Standards for School Mathematics, National Mathematics Advisory Panel, 2008, p. xx).

Gesture in the context of classroom mathematics has been increasingly an object of study due to its power as both a window into the gesturer's mental representations of the mathematics (e.g., Abrahamson, Gutiérrez, & Baddorf, 2012; Hostetter & Alibali, 2008; Nemirovsky, Rasmussen, Sweeney, & Wawro, 2012; Stevens & Hall, 1998) and as an instructional tool to guide learners' focus and representational thinking (e.g., Alibali et al., 2013; Church, Ayman-Nolley, & Mahootian, 2004; Mehus, Stevens, & Scopelitis, 2010; Scopelitis, 2013; Singer & Goldin-Meadow, 2005). These studies have taken many forms, including psychological (for review see Goldin-Meadow, 2003; Goldin-Meadow & Alibali, 2013), cognitive linguistic (Nemirovsky, Rasmussen, Sweeney, & Wawro, 2012), or interactional (Singer, Radinsky, & Goldman, 2008), leading to a rich body of knowledge that convincingly reveals that analyses of learning and instruction cannot ignore nonverbal communicative body movements.

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Color versions of one or more of the figures in the article can be found online at www.tandfonline.com/hcgi.

The current study focuses in particular on the role of nonverbal communicative practices within an instructionally important type of mathematics classroom discourse, instructional analogy. Mathematics is a discipline in which expert-like thinking involves recognizing relationships between concepts, problems, or ideas (e.g., Bransford, Brown & Cocking, 2000; Polya, 1945), and educational reform recommendations regularly focus on the importance of supporting students in drawing mathematical connections (e.g., National Mathematics Advisory Panel, 2008; National Research Council [NRC], 2001). One way of drawing connections is to develop higher order links between representations of mathematical objects. Specifically, one may conceptualize mathematical ideas, problems, or solutions as systems of relationships and then draw connections between these ideas, problems, or solutions and others (see Vosniadou, & Ortony, 1989).

For example, to represent a mathematical object such as a word problem as a system of relations, one could dissect the problem into its component relationships and manipulate them, such as the following simple example: *Joey is 3 inches taller than Susan, who is 2 inches shorter than Mary. How many inches taller than Mary is Joey?* One can easily solve this problem by re-representing the relations into a transitive inference structure using the following logic: Mary must be between Joey and Susan, and she is 2 inches taller than Susan, so she must be 1 inch shorter than Joey. Re-representing this system as transitive inference is especially useful if the student next needs to solve a new problem in which transitive inference is again useful.

After doing two such problems, a teacher can then draw students' attention to the similarity between these problems, in order for students to develop a more explicit, conceptual understanding of what they have done in both of these problem contexts. Thus, analogy in classroom contexts may appear quite different from the formalized "a" is to "b" as "c" is to "d" format described in theoretical treatments, but this nonetheless describes the process of mapping relationships between two relations or systems of relations (i.e., the relationship between "a" and "b" is the same as the relationship between "c" and "d"). A growing body of work has examined analogy in classroom contexts, suggesting that drawing structured relationships may be an important part of higher order thinking in classroom mathematics (see Richland, Holyoak, & Stigler, 2004; Richland, Zur, & Holyoak, 2007; Rittle-Johnson & Star, 2007; Star & Rittle-Johnson, 2009). These comparisons between mathematical representations may be between multiple problems (Kellman, Massey, & Son, 2010; Richland & McDonough, 2010) or between multiple solution strategies (e.g., Richland & McDonough, 2010; Rittle-Johnson & Star, 2007). Studies of the interactions within these teaching moments reveal they are quite challenging, however, and ensuring that students are actually constructing relational representations and then linking them is a complex teaching activity (Stein, Engle, Smith, & Hughes, 2008).

Gesture may play an important role in the way teachers express relations between and among representations in the mathematics classroom (Alibali et al., 2014). These are contexts in which educators are aiming to help clarify for students how problem relations can be identified and mapped together. Gestures are a rich source of information about both the speaker's conceptualization of the information and the speaker's perception of the recipient's communicative needs.

One may take the framing of conceptual blending theory (Fauconnier & Turner, 2002) to conceptualize how the compared representations, as well as the meaning conveyed in gesture, dynamically align and correspond to produce a very rich space for meaning making. While analogical reasoning is conceptualized as a highly organized system of mapping correspondences according to pragmatic, contextualized goals, conceptual blending theory provides more general mechanisms for considering mappings, enabling broader inclusion of gesture (Parrill & Sweetser, 2004).

As highlighted by Fauconnier and Turner (2002), these multiple levels of alignment can lead to a deep conceptual blending of concepts such that there are many simultaneous levels of connection conceptualized at once, described by these authors as *integration networks*. These integration networks build rich interconnections between representations such that the representations are interwoven and manipulated. This may lead to potentially contradictory alignments and integrations of concepts, or aspects of the representations may be compressed, meaning that aspects of the conceptual framings have been collapsed, reorganized, or chunked together.

This process may unfold differently for explicit links and alignments between representations that are relatively new to the reasoner, versus links between representations that are abiding and may draw on deep embodied experiences, such as conceptual metaphors (Lakoff & Johnson, 1980). The theory of conceptual metaphors describes metaphors that are internalized, often early in life, and are almost impossible not to integrate into one's thinking about a target domain. Such metaphors are often only evident by examining slippages within one's speech, such that phrasing from one metaphorical domain is used in reference to another (e.g., using phrases such as "my heart is on fire" to signify the blending of temperature and emotion).

When the analogy is more novel, however, for example between two mathematical equations, the process of discovering the utility of an analogy to link, align, and potentially integrate representations is not automatic. In fact, one of the most challenging parts of leading others to learn through analogy is ensuring that they notice the utility or possibility of comparing one representation to another (e.g., Gentner, Ratterman, & Forbus, 1993; Gick & Holyoak, 1980, 1983; Richland & McDonough, 2010). Often participants show difficulty noticing the usefulness of an analogy, despite the presence of two related analogs in verbal, written, or visual form. At the same time, often the same participants can make and use the analogy once they are made aware of its relevance (Gick & Holyoak, 1980, 1983).

While the framework of conceptual integration provides a strong paradigm for how to understand gesture's role within meaning making as one of providing an additional representational space in mental model formation (Parrill & Sweetser, 2004), the role of gesture in analogy as structure-mapping has not been yet considered. Gestures provide a modality beyond oral speech through which to direct learners' attention to shared representations (Hindmarsh & Heath, 2000) or to support the imagistic elements of language (for treatments of the mechanisms underlying this process, see views by Efron, 1941; Goldin-Meadow, 2003; Kendon, 1994; McNeill, 1992). Thus, gestures produced that align with spoken analogies could be a key tool for teachers to ensure that students notice and benefit from instructional analogies.

This project seeks to extend and deepen a new literature on deictic linking gestures, one key type of gesture used within analogical classroom discourse, and to examine the relations between linking gestures and analogical structure-mapping. To situate these communicative hand and arm movements within the broader categorization schemes of gestures, these can be classified as a sub-category of deictic gesture (McNeill, 1992). As originally posed by Efron (1941) and reformulated by McNeill (1992), deictic gestures reference objects in shared communicative space (such as "this," "that," and "here").

Linking deictic gestures thus would be those that explicitly move between two or more deictic objects in combination with discourse that draws a meaningful relationship between these objects. These gestures have been previously described variously as "linking" (Alibali et al., 2014; Nathan

& Alibali, 2007), or as “comparative” (Richland, Zur, & Holyoak, 2007), but for clarity sake, these will now be referenced in full as *deictic linking gestures*, or *linking gestures* for ease, with the aim to specify that they are hand and/or arm motions that move between shared representations in a communicative context. For example, a teacher who uses the analogy of a line segment with infinitely many points to help explain the numerical concept of infinitely many numbers in an interval might move her hand from a representation of numbers within the numerical interval to their corresponding points on a number line (Vamvakoussi & Vosniadou, 2012). Close analyses of two of such instances are used to demonstrate the range of linking gestures identified in the sample, from swooping linking gestures (Figure 2) to deictic point linking gestures (Figure 3). As noted by an insightful reviewer of this article, both types of links are in some ways iconic of the mental links the teacher hopes to forge in the listener.

While these gestures act in the service of analogies, one might imagine they could be alternatively framed within what have been categorized as metaphoric gestures in several category schemes (e.g., Efron, 1941; McNeill, 1992). While there are metaphoric gestures that also emerge during instructional analogies, metaphoric gestures as defined in these taxonomies use the gesture itself as the metaphor base, while the current work focuses on gesture as a support for comparisons and connections developed between mathematical objects represented outside of the body and physical space. Thus, while metaphoric gestures provide links as well, the current analysis focuses on linking gestures within the category of deictic gestures.

These linking gestures are important to mathematical instructional contexts, because drawing connections is a core element of developing conceptual, flexible representations of mathematics (NCTM, 2000, p. 2; National Mathematics Advisory Panel, 2008; Polya, 1954), but it is notoriously difficult to teach others to accomplish (Hiebert et al., 2003). Thus, gesture that aligns with and embodies discourse that conceptually links representations already provided and instantiated by the instructor may be a powerful tool for supporting mathematics students in productively engaging in connected mathematical thinking. This article draws attention to the potentially key role of this type of gesture and examines its prevalence in mathematics classrooms within the United States and two regions that score more highly internationally in standardized mathematics tests, Japan and Hong Kong, China (Gonzales et al., 2008). Although the cross-cultural paradigm should not be used to draw causal relations between national achievement patterns and classroom practices, cultural variations make visible malleable factors in classroom instruction (Silver, 2009). Systematic differences in gesture use provide insights into teaching practices that might be important, malleable, and cost-effective factors in classroom teaching.

Teachers’ gestures are a readily available, naturally occurring resource for directing learners’ attention to similarities and differences between objects or phenomena. There is growing evidence that teachers’ gestures are very closely tied to learning and cognition, either as a complementary source of information (e.g., Kendon, 1994; Singer & Goldin-Meadow, 2005), as a part of discourse that deepens and extends cognitive resources for engaging with conceptual problem solving through offloading processing resources (see Goldin-Meadow, 2003), or as a motoric medium for embodying mathematical concepts (Glenberg & Robertson, 1999; Hostetter & Alibali, 2008, 2010).

Gestures are known to show cross-cultural differences, with varying normative “vocabularies” of gesture use (Kendon, 2004; Kita & Ozyurek, 2003; McNeill, 2000) and developmental usage patterns (Iverson, Capirci, Volterra, & Goldin-Meadow, 2008). These can impact the recipient’s learning from a speaker’s gestures. For example, Graham and Argyle (1975) found that gesture played a larger role in recipients' ability to mentally represent an object described by Italian speakers than for American speakers. Instructionally, a close analysis of instructional gesture use revealed differences between Japanese and U.S. teachers (Alibali, Sylvan, Fujimori, & Kawanaka, 1997), suggesting that this could be a productive area for future study as a way to better understand cross-national teaching variations.

This article examines the role of linking gestures during a particular discursive context, instructional analogies, that are explicit interactional attempts to link mathematical representations. This project expands and deepens an analysis of cross-national differences in teachers' analogy use in the United States, Hong Kong, and Japan (Richland, Zur, & Holyoak, 2007) within the video data collected as part of the Third International Mathematics and Science Study (TIMSS 1999; Hiebert et al., 2003). In that study, one analysis reported differences between the numbers of linking gestures used by teachers in these three regions. Specifically, U.S. teachers were statistically less likely than teachers in either of the other regions to use linking gestures when they taught instructional analogies. This suggested that the use of linking gestures varied culturally and that there might be a correlation between the use of linking gestures and teacher effectiveness at leading their students to draw mathematical connections. Since both Japanese and Chinese (Hong Kong) students performed higher than U.S. students in mathematics achievement as identified in the TIMSS standardized test data (Gonzales et al., 2008), this could ultimately be impacting student achievement, though that was not tested directly.

While a provocative finding in the context of the rest of that analysis, more analyses are necessary to fully understand cultural patterns of linking gesture usage. First, that analysis did not consider non-linking gestures made concurrently with linking speech, despite the fact that many of those gestures were representational and had relevance to either the source or the target of the linking communication. The current article does examine these gestures, describing them as "within-representation" gestures, since they are clarifying or in some way modifying only one of the representations being linked during the linking episode. The current article provides detailed qualitative examples of both types of gestures in the context of a Japanese analogy, clarifying the distinction between these types of gestures, and the frequencies of both linking and within-representation gesture use are presented together for all three regions. They are also now analyzed by region, to explore the relations between region and type of gestures used.

Secondly, there are known differences in the amount of visual information provided to students during math lessons across countries, such that Hong Kong and Japanese teachers are more likely to make representations visible during instructional interactions than U.S. teachers (Richland, Zur, & Holyoak, 2007). Thus, the reported linking gesture results could be explained by practices of board use, rather than any specific differences pertaining to gesture use, since they did not control for baseline visual information. The current article reports an analysis of linking gesture use only in cases where two or more visual representations were available to be linked.

Finally, a closer analysis of these linking gestures would enable a more detailed exploration of when teachers used linking gestures. A recent study has suggested that linking gestures may be used systematically in relation to the type of information being presented (Alibali et al., 2014). Alibali and colleagues (2014) used a related but different analysis plan from the current study, in which they examined three lessons from each of six U.S. teachers, and coded all episodes in which teachers linked mathematics. Within those episodes, they explored when the teachers used gestures to support the links made verbally. They found that these teachers used linking gestures more often when introducing new material than when reviewing previously taught content. This might indicate that these teachers were calibrating their linking gesture use to conditions in which the links are most difficult for learners—cases when prior knowledge is limited. Limited prior knowledge is known to reduce learning from analogy (Rittle-Johnson, Star, & Fyfe, 2013), so this could be a condition in which gesture use would be particularly important.

Our analysis extends this body of inquiry to examine whether linking gestures during analogies were more frequent in Hong Kong and Japanese versus U.S. classrooms, or whether these teachers across regions used them in ways that were related to their expectation about the students' difficulty of seeing the links within certain instructional analogies. To address this question, and building on the Alibali and colleagues' findings, the current analysis examines whether teachers in these higher achieving regions used linking gestures more often when the source analog was introduced recently—a condition of relative analog novelty that experimental data suggest would be particularly taxing for the students as reasoners. Lack of knowledge about source or target representations is known to make analogies more difficult to complete successfully (Gentner & Rattermann, 1991), such that more instructional support is necessary (Gentner, Loewenstein, & Thompson, 2003; Richland & McDonough, 2010).

The larger study situating the current analyses is next described in more detail, followed by a more full treatment of why linking gestures have the potential to greatly impact student thinking. Transcripts of two sample instructional analogies and their integral linking and nonlinking (within-representation) gestures are used to better define the construct of a linking gesture, and the results of the expanded analyses of teachers' cross-national variations in use of linking gestures are reported.

METHOD

Video Data

Videotaped classroom lessons were randomly sampled from the corpus of data collected as part of the Trends in International Mathematics and Science Study–Repeat (TIMSS-R; Hiebert et al., 2003). The original TIMSS-R dataset was collected as a randomized probability sample of all lessons taught in public, private, and parochial schools throughout the country in one year. Each selected classroom was videotaped on one occasion during a normal class period. Lessons were typically 50 min long, yielding approximately 25 hrs of videotaped data that were examined in this project.

For the current analysis, 10 lessons were randomly selected from the TIMSS-R data collected in the United States, Hong Kong, and Japan, yielding a subset of 30 total lessons. Hong Kong and Japan were selected for comparison to the United States because their students outperform U.S. students on international tests and because their classroom teaching styles are quite different from each other as identified in the primary TIMSS-R findings (see Hiebert et al., 2003).

Observational Coding

This subset of 30 lessons was analyzed using temporally linked transcripts and videos. The codes, methods, and data reported previously by Richland, Zur, and Holyoak (2007) are briefly described here to provide a foundation for the current, new analyses. In that study, qualitative codes were used to generate quantitative frequency data about all instances in which teachers drew explicit links between multiple representations. Three of those codes will be used in the current analysis and are described in more detail below: source visibility, source novelty, and presence/absence of linking gestures.

This study also makes use of the units of analysis identified by Richland and colleagues: relational comparisons and explicit linking opportunities. Lessons were first divided into units of analysis in which all identifiable instances of relational comparisons (also called analogies or instances of explicit linking between representations) were marked. In a series of passes, these relational comparisons were then categorized according to previously designed codes. At least two coders divided the data for each pass, and reliability was calculated for all coders on approximately 20% of the data. All disagreements were resolved through discussion. When enough information was not available to make a decision on a particular code for a specific unit of analysis (e.g., the camera did not capture a section of a worksheet), that relational comparison was excluded from analysis of that particular code.

Identification of Instructional Analogies

Two expert coders (researchers with relevant doctorates) separately identified all units of relational comparisons within every lesson. These might have been one turn or multiple turns, as long as the topic of the interaction pertained to one of the representations being linked, or to the links between the representations. All disagreements were resolved by discussion. The definition of a relational comparison derived from Gentner's (1983) structure-mapping theory of analogy. Relational comparisons were defined as a higher order relationship (either similarity or difference) between the relations within a source object and within a target object. Based on this definition, there were several criteria used to mark an interaction as a relational comparison. First, a source and a target had to have been clearly identifiable. For example, an instance in which the teacher stated, "Let's solve this problem like you used to do them" was not marked because the precise source was not readily apparent to us or to learners. Second, for a positive identification there had to be some clear indication of the intention to compare the source and target items. These could be an explicit verbalization (e.g., "X is just like Y") or a less explicit verbalization signifying a same or difference comparison (e.g., "We just finished with X, now lets do another one [Y]"). Many classroom lessons contained multiple representations that were never directly compared, so these were not coded as relational comparisons.

Teachers in all countries produced a high number of relational comparisons during the 10 eighth-grade mathematics lessons. As reported by Richland, Zur, and Holyoak, (2007), a total of 195 units were identified in the 10 subsampled U.S. lessons, with a mean of 20 and a range of 9–30 per lesson. A total of 185 units were identified in the 10 Hong Kong lessons, with a mean of 18 and a range of 7–27 per lesson. A total of 139 comparisons were identified in the 10 Japanese lessons, with a mean of 14 and a range of 9–25 per lesson. Thus teachers in all regions were frequently drawing explicit links between representations for instructional purposes.

Source and Target Simultaneously Visible

Every unit of relational comparison was then categorized in a binary code for whether or not the source was visible to students while the target was taught. A positive score was given if the source was easily visible to students while they were expected to be reasoning about the target. Generally, this meant that the source was written or drawn on a classroom chalkboard, on an overhead projector, or on a class worksheet and was left in that location while the teacher moved their attention to the target. This code was considered important as a strategy for supporting students' higher order reasoning. Retrieval and working memory demands increase when the source is not available (Cho, Holyoak, & Cannon, 2007). The reasoner must successfully identify and retrieve the source object, as well as manipulate the representation in working memory to determine alignment and structural relationship to the target object. These demands can be considerable if the complexity of the relational structure is high (Halford, 1993; Waltz, Lau, Grewal, & Holyoak, 2000). Processing demands may be further compounded if the source object is novel and the reasoner cannot take advantage of expertise to chunk relations (Chi, Glaser, & Rees, 1982; Kimball & Holyoak, 2000).

Eight units were excluded from analysis due to an inability to code the materials. This occurred when the video camera did not sufficiently allow coders to identify the visual status of the source object. Reliability between coders was calculated as 87%.

As reported by Richland, Zur, and Holyoak (2007), U.S. teachers made the source visible during discussion of the target at the lowest rate across all three countries. U.S. teachers were least likely to provide the students with visual access to the source while making links to the target. Visual representations can highlight relational structure and reduce both retrieval and working memory demands for representing complex relations, so this difference may indicate that U.S. students face higher processing loads than students in Hong Kong and Japanese classrooms.

Source Novelty

Each source analog from every relational comparison was coded for how recently the source had been introduced to the students based upon information explicitly provided in order to gain an index of the teacher's expectation about whether this information was relatively new to students (produced within the same lesson or unit) or was common knowledge due to earlier school introduction (such as arithmetic) or due to everyday cultural schemes outside of school. Coders made this judgment using verbal statements produced during the relational comparison, signifying either a specific time in which the source had been introduced or conveying an expectation that the provided information was common knowledge for the students (although whether it actually was for all students is not known). If no information was available, no code was given. The code was scored first by coders marking whether the source had been presented (a) in the same lesson, (b) in the prior lesson or within the same unit, (c) prior to the current unit, or (d) if the knowledge was expected to be culturally standard common knowledge.

The following are two examples that reveal how the speaker's talk was used to categorize the source. These are both teacher examples. The first was given a score of "1" because the source problem had been given and solved during the current lesson: "*Ok see how our last problem*

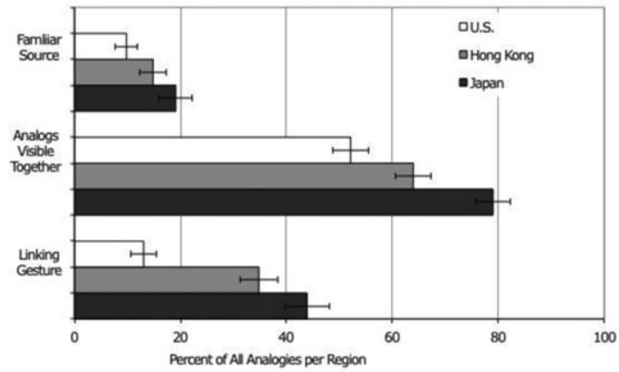


FIGURE 1 Teacher's use of linking gestures and visible source and target representations across all analogies produced in U.S., Japan, and Hong Kong regions (data reprinted from Richland, Zur & Holyoak, 2007).

ended in x = 0, and now we've just solved one with the solution 0 = 0. Let's talk about what the differences in these solutions actually mean." An example of a "4," culturally standard common knowledge, would be the following: *"Imagine you are playing football and you lose some yards, and then you lose some more. The end result is more negative. Adding negative numbers is just like that."* While it is possible that the student in this U.S. interaction does not actually know about football, the teacher's lack of explanation is displaying an expectation that this is shared knowledge.

These codes were then collapsed into a binary code conveying that the source analog was either "relatively novel" (information taught in the current lesson or unit) or "not novel" (standard cultural knowledge and math learned prior to the current unit). Coders who had attended school in each of the three regions, the United States, Japan, and Hong Kong, provided supplemental information about whether a source was a part of standard cultural knowledge for these students and how to interpret the linguistic phrasing provided by the teacher. Additional information about the timing of presentation of the information was available from a lesson plan document in which videotaped teachers were asked about the lesson as a whole and the level of students' prior experience with the topic, as well as where this lesson fit into the sequence of the instructional unit. Because individual pretests or specific individual data on understanding were not available for the students in the specific classrooms for which lessons were analyzed, this code measured the novelty of the source information based on all information visible within the lesson video, rather than through an independent calculation of individual students' knowledge. While this may misrepresent the retention or understanding of some students, the analyses in the current article focus on teachers' gesture use and its relationship to these teachers' demonstrated expectations of their students' prior knowledge.

If sufficient information about source novelty was not provided, the comparison was excluded from subsequent analyses involving source novelty. Eight units were excluded from this code, leaving an overall sample of 511 analogies. Reliability was 90% between coders. Previously, we have reported that there was a significant difference in the use of familiar sources by country (Richland, Zur, & Holyoak, 2007). As shown in Figure 1, teachers in the United States were more likely to use novel sources than either teachers in Hong Kong or Japan. We now seek to understand how teachers' assumptions about the novelty of the source analogs related to their use of linking gestures.

Linking Versus Within-Representation Gestures

Coders first marked presence or absence of any gesture that occurred in the context of talk about the source representation, the target representation, or the relationships between these, within each time period identified as containing a relational comparison. Gestures were defined as hand and arm movements that conveyed information that was relevant to instruction of mathematical content key to the compared representations, rather than affective or interactional content (e.g., pointing at the board would be included, while pointing at a student would be excluded). These could be iconic, representational, or metaphorical gestures that moved in a way that demonstrated some aspect of the mathematical information, or deictic movements that only carried meaning in the context of the interaction in which they were embedded (e.g., pointing to a specific number on the board). Gestures that were unclear as to their intention were not included (e.g., "beat" gestures that moved with speech).

Once gesture was identified within a unit of comparison, all gestures during the comparison discourse were analyzed to determine whether there were any linking gestures. If there were none, the gestures were coded to determine if there were within-representation gestures. Linking gestures were defined as movements that were used to physically instantiate the connection between representations being compared. This generally meant what is being describing as a deictic linking gesture, with a hand or arm moving in a string of deictic gestures without returning to rest at the body between, such that the hand moves deliberately and directly between one representation and the other representation. The movement itself could have followed varying meaningful paths, such as moving in an arc between the representations with an open hand so that attention is directed to two whole representations, directly between two points indicating a more constrained link, or with simultaneous points to the two linked representations. These are described in more detail within the Results section below, with some illustrations in Figures 2 and 3. If such a gesture was identified, the comparison was marked as containing a linking gesture.

Within-representation gestures were defined as iconic, metaphorical, or deictic movements that pertained to one of the compared representations but importantly not both. This could include a deictic gesture that pointed to each number in a source equation as it was being read aloud, or it could include gestures that moved from the problem to a target solution—so two parts of the same problem space. Thus if the gesture(s) were deemed to only reference one or the other of the objects being compared, even if they moved within that representation, the comparison was coded as containing a within-representation gesture.

The linking gesture code took priority over the within-representation gesture code. Any presence of a linking gesture meant that the relational comparison unit was scored as having linking gesture. If there was no linking gesture, the unit was scored as either having within-representation gesture or not having any gesture. All relational comparison units in which the video camera or teacher's body angle did not allow for clear interpretation of the gestures were excluded from analysis. This led to 12 exclusions, leaving 507 units for analysis. Intercoder reliability was 91%.

A

T: よこがけさんは[10個だったから調べれただったけれども、
Hara [could figure it out because it was only 10 cakes,
[*((uses chalk in right hand to point to (touches) Hara's solution on the board where he calculated costs for ten cakes, then sweeps down over the whole solution))*

T: もしかして、これ、[2つのケーキを合わせて100個買いなさいって言われたならば、
but if this were to say
[combine these two kinds of cakes and buy 100,
[*((uses chalk to point to (touches) the 10 in the word problem written on a sheet of paper pasted to the board))*

((some lines omitted))

T: ところが、りか[がやったこういうやり方でいくと、答えがすぐポンと、こう出てくる。
However, if you go with
[this kind of process like Luiko used, the answer comes right out.
[((right hand points to the full inequality equation written on the board as part of Luiko's solution: 230x - 200 (10-x) < 210))
T: だから、[一個ずつ数を調べなくてもいいから、
So [because you don't have to figure them out one by one,
[*((left hand makes open palm gesture with three beats))*

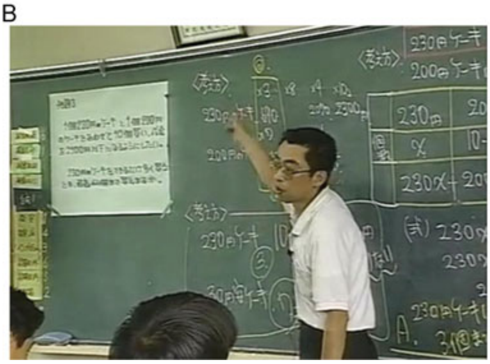
T: [不等式作って、
[making an inequality
[((right hand touches Luiko's inequality solution with pencil - 1 in Figure 2C))

T: [解いていくことって、
and [figuring it out from there
[((hand moves continuously over Luiko's work with pencil - 2 in Figure 2C))

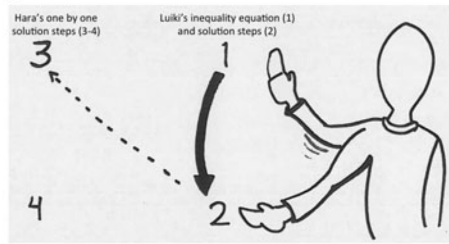
T: [一個ずつ数えるのよりも随分良いところがあるよ。
has a [bunch more positive facets than doing it one-by-one.
[((moves his right hand to Hara's solution in a continuous motion and circles the written steps used to find his answer 3 and 4 in Figure 2D))

T: っていうことでございますよ。
It's that sort of thing.

B



C



D

FIGURE 2 Swooping Linking Gesture. Transcript shows a Japanese teacher comparing two solutions using a swooping gesture and an accompanying diagram to highlight the benefits of using an inequality equation (2A). A photograph of the board shows both solutions, written for the whole class to see (2B), and a sketch (2C and 2D) shows the teacher's movements from Luiko's inequality equation and solution to Hara's one-by-one, trial-and-error solution. The linking gesture (bolded in 2A) moves down over one solution method (moving with a hand holding a pencil from the space listed as "1" to space "2" in Figure 2C), down momentarily and without pausing, moves continuously to swoop from the first to the second solution strategy (from "2" to "3" in Figure 2C), and down over the second solution (over "3" to "4" in Figure 2D). Video viewable at: http://timssvideo.com/53, from 33:55 to 34:30.

RESULTS

Two types of data analyses are provided. First, excerpts of two instructional analogies are analyzed, both of which include linking gestures, in order to explain the range of linking gestures identified in the video corpus and to deconstruct the relations between these gestures and the discursive linking context in these two cases. The first is a segment of a lesson produced by a teacher in Japan who compares two student solutions in order to highlight the greater utility and generalizability

A

Teacher: I [multiplied this[A] times this[B] and divided by this[1]

**[*((Points to each relevant number in the first*
row of a table. Without a pause, moves hand and
repeats the pointing pattern with the focal table row,
showing the relationships are the same in both rows))

I multiply this[C] times this[D] I divide by…

((holds pointing finger out and then after a brief
pause brings it down to the bottom number))

(.02)

Student: One hundred twenty six[@]

NOTE: *Superscript denotes the object of the point as symbolized in Figure 3B.*

B

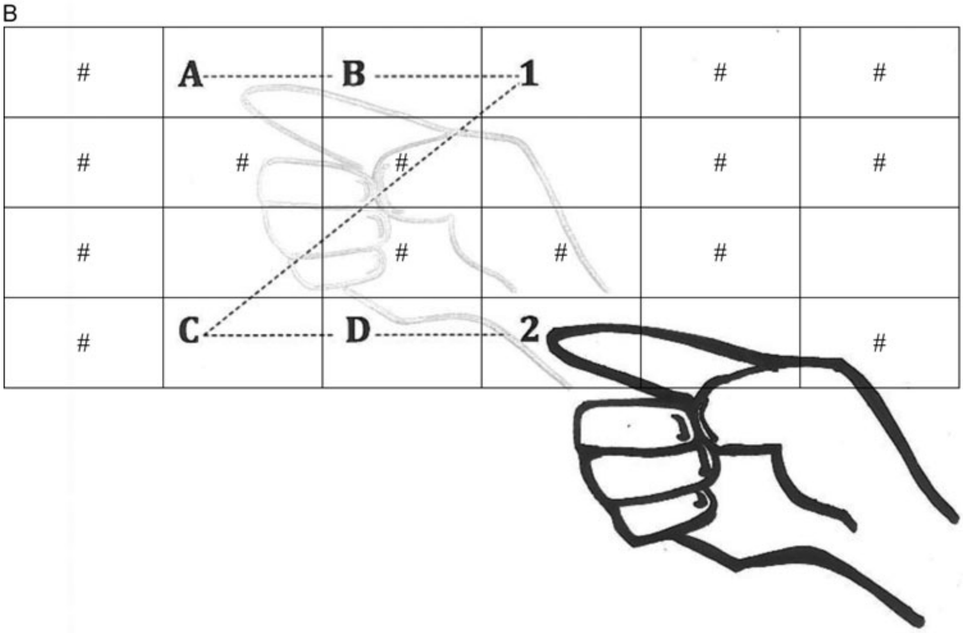


FIGURE 3 Deictic Point Linking Gesture. Transcript of a U.S. teacher drawing links between a prior problem and a current problem. As represented in Figure 3B, his finger moves between precise parts of a table in time with this speech, "This" simultaneous with "A", *divided by this* "B" *multiplied by this "1."* Without a pause, the teacher's finger moves to the second line and the pattern is repeated in the same rhythm, moving from "C" to "D" to hover over "2" before pointing to it.

of the more efficient solution. The second is a segment of a U.S. lesson in which a teacher uses a linking pointing gesture to tightly constrain students' interpretation of his verbal links. Following these qualitative analyses, quantitative analyses are provided that reveal variations in the relationships between teachers' linking gestures and the interactional context. Specifically, U.S. teachers seem to use fewer linking gestures to aid verbal links and vary their gesture use less reliably in relationship to the novelty of the verbalized analogy than teachers do in Hong Kong or Japan.

Qualitative Analyses

Excerpt 1: Linking versus within-representation gestures in a Japanese lesson. Figure 2 provides an English translation of a rich example of linking and within-representation gestures produced in the context of a Japanese comparison between two students' solutions to a problem. Many Japanese lessons within this sample followed the same pattern of providing a complex problem to students who take some class time to solve it, and then two or three solutions are written on the board. The teacher then summarizes and compares these strategies. This figure reveals two types of linking gesture movements. First, it reveals a swooping movement, with a hand that is held open, with fingers loosely outstretched, moving in a continuous arc between the two representations, suggesting that the viewer attend to the more full representations and everything that is covered by his swooping movement (see Figure 2A, Lines 34–44). Second, this excerpt illustrates a second common version of a linking gesture, which is a simultaneous movement with both hands, such that one hand was directed toward one representation at the same time that the other hand was directed toward the compared representation (see Figure 2A, Lines 30–35). Since both hands are actively engaged, outstretched from the body's core, and draw the listener's attention, these are coded as linking the two representations.

In this classroom excerpt, a teacher previously had asked all students to consider the following scenario: A boy needs to purchase 10 cakes using a specified amount of money, with as many cakes as possible being the more expensive of two kinds. If he purchased all 10 of the expensive cakes, however, he would be over-budget. How many of the expensive cakes could they purchase?

As is common in Japanese classroom lessons (Stigler, Gonzales, Kawanaka, Knoll & Serrano, 1999), the teacher invites two preselected students to the board, where they describe their solution strategies, and then he asks all other students if they used these strategies (see Figure 2B for a photograph of the board in which both solutions are written for the whole class to see). One student uses a strategy that works accurately but that requires a slow one-by-one process of trial and error to find the right combination of numbers for each of the two cake types. In contrast, a second student developed a very efficient inequality equation, which can be used on any range of numbers and quickly provides an exact solution. In a mode that is common across the corpus, the teacher co-constructs a final comparison between the two solution strategies with his students, and his gestures during this comparison are illustrative of other Japanese teachers' linking and within-representation gesture use. The first student solution presented was provided by Hara, who calculates the cost for 10 of the expensive cakes, then nine of those and one of the less expensive kind, then eight and two, and so on. The second student solution (Luiko's) develops an inequality equation, which can solve the problem more efficiently. Following a discussion of the strategies, both of which prove correct, the teacher writes on a side board (translated into English): “Let’s understand the benefits of finding the answer by setting up an inequality.”

Following this introduction, in his summary of the problem and the solution strategies that have emerged, the teacher first uses within-representation gestures to focus students’ attention on key properties of each of the visual representations of the solutions while describing them. In Lines 2–5 (see Figure 2A), he uses a pointing gesture to highlight the same part of the problem solution he is emphasizing verbally, namely, the fact that Hara’s one-by-one trial-and-error strategy worked for calculations involving a small number like 10 total cakes. Because he is pointing in parallel with his speech, and the speech is entirely focused on one representation (solution one), this is defined as a within-representational gesture rather than as a cross-representational linking gesture.

He continues this episode by pointing to the number 10 in the original problem (Lines 1–10), while he poses a hypothetical scenario (and creates a link to another, hypothetical, mental representation, though that is not the unit of analysis being focused on here) in which the problem had asked for 100 total cakes. This is a second analogy unit, now between one representation (the 10 cakes problem and Hara’s solution) and a new mental representation (the 100 cakes problem and an imagined solution). There is no visual representation of the 100 cakes or its solution, nor does the teacher use gesture to create a substantive representation of the 100 cakes problem. Instead, he points only to the visible representation of the 10 cakes problem (the first representation). Therefore, this is again defined as a within-representation gesture not a linking gesture.

This within-representation gesture is an important gesture for the students’ meaning-making. Though focused only the visual information present in the problem and Hara’s solution, this gesture emphasizes to the students that they must change and reimagine their understanding of this solution in the context of the new hypothetical problem. The gesture also helps direct their attention to what features must be prominent in their mental representation of the target (new problem) representation. However, it does not have the full potential of a linking gesture that moves between visual representations to draw students’ attention to common features of the shared materials.

If we return to considering the impact of these within-representation gestures to the initial comparison of interest, between Hara’s and Luiko’s solutions, these pointing gestures within the source representation support the students in elaborating their mental representations of Hara’s solution by emphasizing the key structural challenge of this one-by-one solution strategy. To restate another way, this teacher facilitates students’ re-representation of the source in a way that will facilitate the forthcoming comparison. He uses gesture to support his verbalized description of the strategy used with the 10 cakes and then emphasizes that this works particularly with the affordances of the small set size of the original problem. Thus, even within-representation gestures have a key role to play in the interactional context of an instructional analogy.

At the same time, the most challenging part of doing analogy is noticing the relevance of drawing a link between two representations, and linking gestures play a specific role in both embodying those linkages themselves, as well as drawing attention in particular to the elements of the representations that should be linked. Thus, they support acquisition of the links between these representations, rather than helping to facilitate representations of the source or target objects. In this episode, the teacher uses linking gestures as a tool to emphasize the relationship and distinction between the two offered solutions. He first states this in a broad way, using the simultaneous linking gesture in Lines 21–31 (see Figure 2A). The teacher seems to run his hand down over Luiko's solution steps (camera does not fully capture this) but ends by using his right hand to hold his pencil in a point at Luiko's final solution to problem (Line 25–27), saying that if one uses this solution, "the answer comes right out" (translation). While holding this point, the teacher then raises his left hand and uses an open palm movement in the direction of Hara's one-by-one solution, moving this left hand rhythmically up and down in three beats as he makes the contrast that this means one "doesn't have to figure them out one-by-one" (translated). The beats seem to be emphasizing the repetitive nature of how one must solve these types of problems when one uses trial and error to test each possible number. Thus, this linking gesture not only draws students' eyes to the two representations (solutions) being compared, but it also provides information about the kind of information the students should be using to differentiate between these representations—despite both leading to correct answers.

In the linking gesture illustrated in Figure 2, the teacher begins the movement in transcript Lines 25–27 (see Figure 2A) when he uses his right hand to point to Luiko's inequality equation while stating this solution will allow you to come to a solution quickly (Lines 28–31), moving his hand over the inequality solution as a whole (Lines 36–39). Finally, in Lines 28–32, he completes this linking with one right hand movement that goes directly from a point to a part of the inequality solution, in which they re-represented the original problem into an inequality statement, to the calculations used to generate a solution in the first strategy (Lines 40–44). In Figure 2C and 2D, this is represented as moving from Luiko's inequality equation (space 1) and down through the steps used to solve the equation (from space 1 to space 2), dropping momentarily without stopping and moving to Hara's one-by-one, trial-and-error solution beginning at the top (space 3) and down over and around the trial attempts to space 4).

This linking gesture deliberately draws the classroom students' attention to the key comparison. Together with the teacher's explicit discourse, these careful gestures will ensure that few students in the class at minimum will fail to notice that the teacher has intended for them to consider the relationship between the two solution strategies. Although no outcome data are available to assess these students' learning, laboratory studies suggest that the primary challenge to learning from analogies derives from the learner not realizing that they should be using one analog to understand another analog or a larger schema (Gick & Holyoak, 1980, 1983; Richland & McDonough, 2010).

Excerpt 2: Linking gestures as constraining points. Linking gestures can also invoke more specific points, as in the deictic pointing linking gesture shown in Figure 3. These types of links may support broad comparisons, as in Figure 2, but they may play a role in more mundane, problem–solution-focused comparisons as well. Figure 3 provides an example of a linking gesture in the context of a U.S. instructional analogy in which a teacher is seeking to help students draw on a prior problem dividing numbers presented in a table to determine how to solve a new table problem. This exemplar reveals another form of linking gesture in which a teacher's hand moves directly between two points, suggesting a much more constrained link between specific parts of two representations (see Figure 3A for the transcript).

The students are solving problems on a worksheet at their desks in which a table has been set up with a variety of foods, numbers of calories, fat, calories per fat, energy needs, and other information is written in columns. The students needed to fill in blank spots within the table by making calculations between the provided data. The current interaction happens after a student requests the teacher's help with calculating the answer to a particular blank spot. The teacher responds by first using a pointing deictic gesture to illustrate the pattern sequence that had been used to fill a prior blank, shown pictorially in Figure 3B, and echoed in speech as well (Figure 3A, transcript Lines 1–4): “I multiplied this times this and divided by this.” He uses a pointing pattern to identify which numbers were divided in the prior problem (“a” and “b” in Figure 3B) to reach the solution “1.” This gesture moves in time with the speech, highlighting aspects within the source representation (coded as a within-representation gesture in the quantitative analysis) to highlight the key elements of that previously completed problem.

Importantly, the use of deictics in both speech and gesture serve to draw attention not to the specific details of the representation but rather to the spatial organization and abstract relations within the table rows. The mathematics itself in these activities is trivial, but the key insight the teacher is attempting to draw is to show students the pattern that repeats within each line of the table, which they can then use to draw inferences about how to calculate a number for each blank space. Specifically here, the teacher creates a predicate structure of the source calculation (A multiplied by B divided by 1) that could also apply to the target calculation. The teacher used the verbal deictic “this” and deictic point gestures which, being deictic, transfer easily to reference the target row. This allows the constructed representation of the first row to be very easily mapped and aligned to the row in which the student had elicited help. Presumably the less embedded nature of the highlighted pattern would then support the student in transferring to other subsequent rows in the future.

Supporting that alignment and mapping, the teacher then moved without a pause from the source points along with speech “this times this divided by this” to a new set of points in the target row. This linking gesture clarifies that the points on the prior problem should be brought to bear on this row. The teacher then uses the same gesture and speech pattern (see transcript Lines 5–7, Figure 3A) but with the points drawing the student’s attention to the next row: “I multiply this times this I divide by . . . ” The speech ends with a designedly incomplete utterance (Koshik, 2002), and the teacher’s finger point lingers in the air, signaling that the student should be coconstructing the mapping, before pointing to the last, key number, which completes the goal of this analogy. The student is left to make the calculations and complete the problem.

While this is a linking gesture that highlights information that is trivial mathematically, it is an effective way for the teacher to draw the student’s attention to the key pattern within the source and to highlight the higher order similarity between the within-representation relationships across the two problem rows. The source row problem had been completed very recently so the procedural routine was likely not yet well encoded to the students, so this linking gesture may have been more important to help students draw on the prior problem than in a similarly procedural mapping where the students had more experience solving problems of this sort.

The subsequent quantitative analyses build upon these identified gesture patterns. Analyses examined the total quantity of gestures produced, along with a better measure of the frequency of linking gestures than had been previously reported in Richland, Zur, and Holyoak (2007). Specifically, frequencies of linking gestures were now examined only in the subset of analogies in which both source and target analogs were visible simultaneously. Finally, relationships between gesture usage and the recency of the source analog were assessed. These allowed for determining whether teachers tailored their gestures to the presumed novelty of the comparison. Source recency was used to examine teachers’ gesture patterns since expertise is known to facilitate attention to relational structure, so teachers might determine that gestures were more necessary with greater perceived novelty (Chi, Feltovich, & Glaser, 1981), which has been found in U.S. samples (Alibali et al., 2014).

Quantitative Analyses

The two levels of gesture codes were analyzed separately to examine any international differences.

Presence or Absence of Gesture

Teachers in all three countries used gesture in the majority of the instructional analogies they produced (mean percentage of all comparisons containing gesture: United States = 83%, Hong Kong = 90%, Japan = 90%). Thus, gesture was a very frequent part of instruction during the segments of the lesson we analyzed. Accordingly, there was no statistical difference in the rates of using gesture of any sort within instructional comparisons across countries, X^2 (2) $= 4.9$, $p = .09$. Very few comparisons were produced without gesture. Note as well that these data underrepresent the total number of gestures during instructional comparisons, since gesture was only coded once per comparison as present or absent. If more than one gesture was produced per comparison (and anecdotally, this was common), this was not recorded. This finding supports and extends other predictions about the ubiquity of gesture across cultures and within instruction in particular. Specifically, it suggests that gestures are a regular part of mathematics instruction in these regions and, further, are common when teachers are conveying informational links.

Linking Versus Within-Representation Gestures

There were differences in the types of gestures that were common across these three regions, particularly when comparing rates of linking versus within-representation gestures. We first examined an overall difference between uses of linking gestures versus uses of within-representation gestures collapsed across country. We found that overall the majority of gestures produced were within-representation, directed toward either the source or target objects individually, rather than linking between the multiple representations: X^2 (1) $= 75.3$, $p < .001$. Thus, most comparison episodes did not include linking gestures.

While fewer, the subset of linking gestures is of great interest, because these are theorized to have the greatest potential to support comparative higher-order thinking. In our previous publication, we reported that the likelihood of using gestures that explicitly linked source and target objects differed by country X^2 (2) $= 36.0$, $p < .001$. Teachers in the United States produced linking gestures in a far smaller percentage of all comparisons than teachers in Hong Kong or Japan (United States = 13%, Hong Kong = 35%, Japan = 44%). Thus, Japanese teachers were more than three times as likely to use linking gestures within a comparison as U.S. teachers.

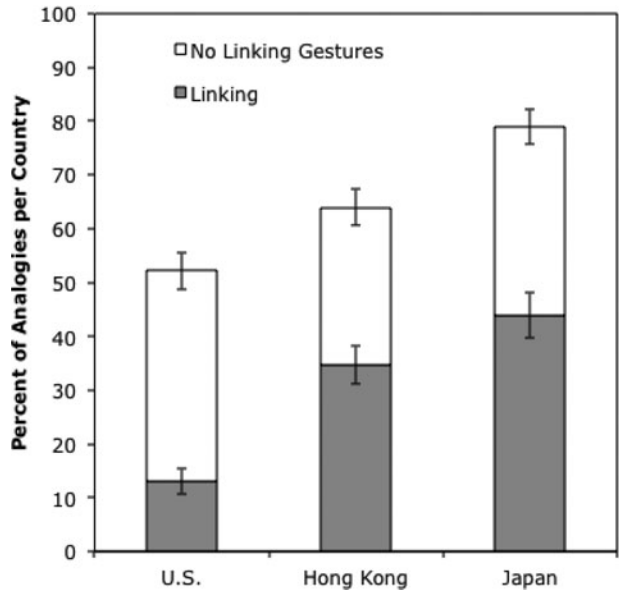


FIGURE 4 All identified relational comparisons in which the compared representations were visible simultaneously, separated into those with linking versus no linking gestures.

That analysis included all identified analogies, however, which necessarily meant that any analogies in which both source and target visual representations were not visible could not be coded as having a linking gesture. A better analysis, therefore, is one that focuses only on analogies in which both source and target analogs were visible simultaneously, since international variations in board use are well documented (e.g., Hiebert et al., 2003; Stevenson & Stigler, 1992).

Thus, a new analysis is here reported to provide a stronger test of the relationship between country and use of linking gestures. This statistic only examined use of linking gestures within the sample of analogies in which the source and target objects were visible simultaneously. This reduced confounding between teachers' uses of visual supports and uses of linking gestures. In this subset of all comparisons, there remained a significant difference in rates of using linking gestures across countries, X^2 (2) $= 23.5$, $p < .001$. As shown in Figure 4, U.S. teachers made the source and target analogs visible together in fewer analogies than either Hong Kong or Japanese teachers. Then, when those analogies were further separated into those with linking versus only nonlinking gestures, it is clear that U.S. teachers were least likely to use linking gestures, even when they had done work to make both source and target analogs clearly visible.

When the individual teachers' data were examined separately within each three regional samples, the pattern remained similar. As shown in Figure 5, there was variability across teachers within all three regions, but the U.S. teachers were as a whole numerically less likely to use linking gestures than their international peers.

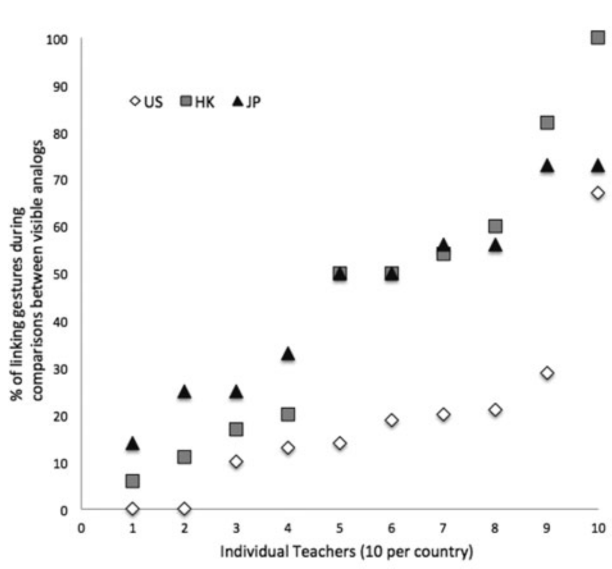


FIGURE 5 The percentage of linking gestures used by individual teachers are plotted for all analogies in which multiple visible analogs are being compared, aligned from teachers with the smallest percentages to the highest.

Gestures and Source Familiarity

The next analysis examined the relationship between teachers' gesture use and the novelty of source analog. Alibali and Nathan (2007) have argued that teachers may use more gestures when topics are less familiar and more abstract for learners. In order to investigate this hypothesis within this sample, rates and types of gestures were analyzed in relation to students' expected familiarity with the source analogs.

First, data for all countries were examined together to determine whether there were common patterns in the relationships between gestures and source familiarity. Overall percentages of comparisons with gestures in which the source was coded "familiar" were compared to gesture rates in comparisons in which the source was coded "not familiar."

There was an overall difference X^2 (1) = 6.89, $p < .01$, such that when analogies were not separated by country, teachers overall used more gestures of any type when the source was more novel. However, there were no reliable relationships between the use of linking gestures and source novelty, X^2 (1) = 2.2, $p = .14$. At least, there were no differences when all countries were examined together.

In order to better understand the patterns by country, data for each country were also analyzed separately. Means are available in Table 1. When examined alone, U.S. teachers showed no reliable patterns of either gesturing more based on source novelty, X^2 (1) = 1.1, $p = .30$, nor of using linking gestures differently depending on source novelty, X^2 (1) = 0, $p = .99$. In contrast, Japanese teachers' use of gesture frequency was related to the novelty of the comparison. They gestured at higher rates during comparisons with more novel sources, X^2 (1) = 7.4, $p < .01$.

TABLE 1
Cross-Cultural Differences in Teachers' Gestures Depending on the Familiarity of Source Analogs

	Gesture present		*Linking gesture*	
	More familiar	*Less familiar*	*More familiar*	*Less familiar*
United States	75%	84%	13%	13%
Hong Kong	83%	92%	11%	38%**
Japan	76%	94%**	37%	42%

**Differences were significant at the .01 level.

There was no difference in their use of linking gestures based on source novelty, X^2 (1) = .46, p = .55, perhaps because Japanese teachers used relatively high rates of linking gestures in all comparisons. Hong Kong teachers, in a different pattern of tailoring gestures to learners' familiarity, did not show a difference in the overall number of gestures used according to source novelty, X^2 (1) = 1.8, p = .18, but they did use linking gestures at a higher rate when source analogs were more novel X^2 (1) = 5.5, p = .01. These data reveal that both Japanese and Hong Kong teachers manipulated their gesture use in a reflection of their perceptions of learners' prior experience with source analogs, while U.S. teachers did not.

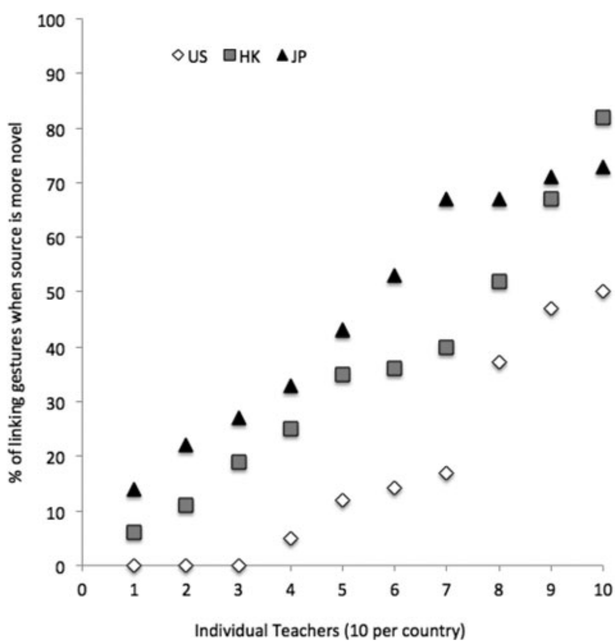


FIGURE 6 The percentages of linking gestures used in analogies coded as more novel are plotted by individual teachers, aligned from teachers with the smallest percentages to the highest.

When examined at the level of individual teachers, again the pattern identified in the overall means is represented across the range of teachers within each country as well. Figure 6 plots the frequencies of individual teachers' use of linking gestures during analogies rated as more novel. The data reveal that there is variability among teachers by nation, but that this variability is less than the overall variability across countries. While some U.S. teachers use a higher rate than others in all regions, overall it is clear that U.S. teachers within our sample use these with less frequency than sampled teachers from either Hong Kong or Japan.

DISCUSSION

Several meaningful results emerged from these data. Least surprising, but useful as documentation, was the finding that all teachers commonly produced gestures during instructional analogies. This is not unexpected, since many studies have documented that teachers frequently use gestures during mathematics instruction (e.g., Alibali & Nathan, 2007; Church, Ayman-Nolley, & Mahootian, 2004; Flevares & Perry, 2001). However, the very comparable rate of gestures during instructional analogies across these three international regions provides an additional window onto the commonality of gesture as an instructional device. In particular, during instructional analogies aimed at promoting abstract, higher order, relational thinking, teachers across all three studied countries seem to naturally integrate gesture with speech directed toward elaborating and helping students re-represent mathematical objects such as problems or concepts. These were the most common type of gestures used in all regions.

The article also deepens our understanding of the role gesture can play in helping students to draw connections across mathematics. Since the importance of making these links is an oft-repeated educational goal, yet one that is difficult for teachers to implement (Hiebert et al., 2003; Stein, Engle, Smith, & Hughes, 2008), linking gesture may have a key part in modeling and facilitating students in meeting this aim. Figures 2 and 3 reveal ways that linking gestures can provide support for students' formation of analogical connections. In Figure 2, the Japanese teacher used within-representation gestures to direct students' attention to key elements of the representations, and then used linking gestures to support deep conceptual links and re-representation.

Similarly, in the U.S. example of Figure 3, the teacher again uses gesture in the first lines to get students to re-represent the first problem as a sequence of numbers in a specific way. In that example, she then uses linking gestures to ensure that students develop a mental representation of the new problem in exactly the same way as the first analog, so that it was quite clear which sequence of numbers should be used again. While not conceptual in the way that Figure 2 is, Figure 3 displays an interconnected use of gesture and relational language that becomes an effective support for students' problem solving. Figure 2, however, is a more substantive example of how linking gestures may play a powerful role in instantiating and reiterating the relationships highlighted within a spoken analogy between two or more representations. The learning opportunity for students participating in the interaction described in Figure 2 is clearly defined by the teacher and instantiated in writing, speech, and in nonverbal linking gesture use. It is important to note that these examples should not be taken as illustrative of cross-cultural differences in the mathematical quality of analogy or linking gestures produced across these three regions, because the analogies were not systematically coded as such, but they provide insight into the broad range of pedagogical goals that can be supported through linking gestures.

International differences were further explored in the frequencies of linking gestures used across these three regions and in the relations between instruction and gesture use. First, a stringent test supported the findings from a less conservative previous analysis suggesting that there were international variations in the use of linking gestures across the three studied regions (Richland, Zur, & Holyoak, 2007). Teachers in the United States were less likely to use gestures that physically connected the two or more representations being compared in an analogy, even when those representations were visible simultaneously, than either Hong Kong or Japanese teachers.

A second set of analyses showed that there were international differences in the relationships between gesture use and the perceived novelty of the source used in the instructional analogy. U.S. teachers did not show any patterns of differentiating their gesture use depending upon the novelty of the source analog. In contrast, both Hong Kong and Japanese teachers varied their gestures depending on the context. Japanese teachers used overall more gestures when making an analogy in which the source was more novel, and teachers from Hong Kong used higher rates of linking gestures. Since expertise and familiarity with a source analog is well known to improve reasoners' likelihood of attending to relational structure (Chi, Glaser, & Rees, 1982; Chi & Ohlsson, 2005; Holyoak, Junn, & Billman, 1984; Novick, 1988; Novick & Holyoak, 1991), novices would be expected to require more assistance in analogies involving more novel source analogs. Thus, these cultural differences in gesture may have real impact on the classroom students' ability to attend to and learn from comparisons between relational structures.

These data differ from the finding that a close analysis of gestures used during a slightly different type of instructional unit—linking episodes—within a small sample of U.S. teachers revealed higher linking gesture use during more novel instructional contexts (Alibali et al., 2014). While seemingly at odds with this cross-cultural analysis, this distinction may be best interpreted as that U.S. teachers may use linking gestures more often with higher task novelty, but they still do modulation of gesture use at a lower rate than teachers in these two higher achieving regions. Also, higher quality teachers may be more likely to modulate their gesture use in such ways.

With increasing reforms, there is growing evidence that U.S. teachers are creating rich opportunities for student learning but are not yet all capitalizing on these opportunities. Beyond gesture, analyses of other ways of supporting instructional analogies revealed that U.S. teachers provided less support for students' relational thinking than Hong Kong and Japanese teachers (Richland, Zur, & Holyoak, 2007). These included use of visual representations, creating a visual record of instruction, and imagery.

More broadly, the larger coding analysis of video data collected as part of the TIMSS-R 1999 video study (Hiebert et al., 2003) showed again a corresponding pattern. While very few of the many coded characteristics of classroom lessons correlated with achievement, the one set of codes that was most informative related to teachers drawing connections between mathematics facts, concepts, or procedures. In the original coding, every lesson was examined to identify all math problems solved or discussed. Each problem was then coded in many ways, including for its purpose as indicated by any problem–goal statements. Problems were marked as making connections, stating concepts, or using procedures. The problems were then coded a second time to measure how the problem was actually discussed and/or solved within classroom instruction to determine whether learners were supported in performing the intended instructional goals (i.e., making connections, defining concepts, or using procedures).

Hiebert and colleagues (2003) found relatively similar frequencies of problem use intended to help students draw connections between mathematics facts, procedures, or concepts across all participating countries—between 13% and 24% of all analogies per country, though Japan had a larger percentage at 54%. A second code examined the classroom interactions through which the problems were actually taught, and this revealed a different pattern. The U.S. teachers were substantially less likely to teach the problems in a way that supported learners in drawing the intended connections than teachers in other regions. In fact, 0% (rounded) of the problems identified as drawing connections were taught in a way that ensured those connections were made during class instruction or discussion. In contrast, between 37% and 52% of the problems intended to draw connections were coded as having done so in the highest achieving countries. The next slightly higher achieving country next to the United States was Australia, who showed 8%.

Teachers in the United States, therefore, created problem-based opportunities for their students to draw connections within the curriculum at a rate comparable to higher achieving countries but did not construct the instruction in a way that ensured that the learners noticed and benefited from the connections. Likewise, in the more specific analysis of analogies between multiple representations during instruction, U.S. teachers produced analogies at a rate comparable to higher achieving countries but did not provide supports in the form of visual and spatial cues to cue learners to the relational commonalities (Richland, Zur, & Holyoak, 2007). The current results further clarify the emerging conclusion that U.S. teachers are invoking high quality, connected learning opportunities for their students but are not supporting students in making these connections, while their international counterparts are doing so.

As a whole, however, the importance of the cross-cultural results is not to argue that U.S. teachers should seek to implement the Japanese or Hong Kong patterns of gesture use (see Silver, 2009) but rather to draw attention to this key feature of classroom mathematics that has been understudied. More importantly, linking gestures and instructional analogies are not likely to be part of teachers' explicit lesson planning or under their direct consideration during teaching interactions. More thoughtful consideration of this interactional tool might provide leverage for teachers to support students in connecting mathematical representations.

Since teaching is known to be deeply integrated with cultural norms and practices, these patterns of gesture use may reflect differing cultural models of normative nonverbal behavior while teaching between the Asian countries and the United States. Variations in teaching within countries are much smaller than variations in teaching between countries, and each country studied in the TIMSS video surveys have revealed consistent teaching routines by country (Hiebert et al., 2003; Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999). Gesture use therefore may be integrated into those routines, particularly concerning how connections between curriculum content are instantiated and supported.

Alternatively, the gesture rates may operate as a window into the teachers' thinking. Gestures are well known to provide insight into the gesturer's cognitive state, beyond what is available in verbal discourse (e.g., Abrahamson, 2007; Alibali, Bassok, Solomon, Syc, & Goldin-Meadow, 1999; Church & Goldin-Meadow, 1986; Roth, 2001). Thus, differential rates of linking gesture use across teachers may reflect either variations in teachers' mathematical content knowledge for teaching and, in particular, differences in how their own mathematical knowledge is linked and interconnected. Cross-cultural variations in teachers' mental representations of mathematics have been documented, and these differences are partly explained by variations in whether mathematics is conceptualized as a serial list of content or as a set of interconnected concepts (Ma, 1999). More connected representations across mathematical content within teachers' knowledge may lead to more linking gesture use.

Teachers who use more linking gestures also may be more sensitive to the conceptual and processing challenges inherent for domain novices to notice and draw connections between mathematical representations. Thus, the differential rates of usage might reveal teachers' ability to assess students' understanding as an instructional analogy unfolds, since presumably students would reveal some evidence that they were not making the unsupported connections. Neither students' behaviors or outcome learning measures were available from this video-survey, however, so such an interpretation is speculative. Beyond students' interactional cues, the Hong Kong and Japanese teachers may have implicit understanding of the empirical finding that less familiarity with source and target analogs leads to more difficulty with noticing and processing higher order relations. Their gesture rates could be indicative of teacher knowledge that more directly aligns with the cognitive psychological research literature regarding how to promote novices' learning for abstract, conceptual knowledge via analogy.

In summary, linking gestures have the potential to play an important role in developing connected, coherent mathematics instruction. Linking gestures during instructional analogies can remind students of the relevance of a source analog to facilitate solving a target, using broad swooping linking gestures, and can integrate verbalized connections between representations into physical correspondences. They can also highlight specific parts of representations that should be brought into alignment, as in the deictic pointing linking gesture shown here.

U.S. teachers are at present not capitalizing on this readily available support strategy at rates comparable to teachers in Hong Kong and Japan. Asian teachers used more linking gestures overall and were more likely to modify their gesture use depending upon their students' level of experience with the source representation. The explanation for these differences is not clear, since they may reflect teacher knowledge, culturally normative patterns of teaching interactions, or variations in teachers' ability to assess students' thinking and needs for cues and support. Regardless, the strong relationship between gesture and learners' mental representations of abstract constructs indicated from the broader literature suggests linking gestures may be a crucial and as yet underoptimized resource for improving the coherence and interconnectedness of U.S. classroom mathematics instruction.

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