

Visual Support for Instructional Analogy: Context Matters

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Abstract

Instructional analogies can overload children's executive function and working memory resources (see Richland, Morrison & Holyoak, 2006), though structure-mapping lies at the core of recommended pedagogy in mathematics instruction (National Mathematics Panel, 2008; NRC, 2001). Videotaped mathematics instruction was manipulated to test the role of visual representations in instructional analogy. Pretest, posttest, and delayed posttest measures assessed 11-13 year old children's learning from one of three versions of the same lesson in which three solution strategies (one a misconception) were compared. Analogs were either a) Not Visible (NV) - presented only orally, b) Partially Visible (PV) - only the most recent solution was visible, or 3) All Visible (AV) - all solutions were visible throughout the instruction. Overall, AV students experienced greater learning gains in procedural knowledge, procedural flexibility, and conceptual/schematic knowledge compared to PV students. These results persist after one-week delay. Apart from procedural knowledge, the same trend is evident when comparing AV students' to NV students' immediate learning gains. Overall, visual representations of analogs within an instructional analogy appear to support schema formation only when they are all visible simultaneously and throughout structure-mapping. Showing students visual representations of analogs but not enabling them to be simultaneously visible led to the lowest performance overall, suggesting this may lead to more object-level encoding than schema formation.

Keywords: analogy; comparison; mathematics education; video stimulus; misconception; executive function.

Comparing different student solutions to a single instructional problem is a key recommended pedagogical tool in mathematics, however the cognitive underpinnings of successfully completing this task are complex. Students must represent the multiple solutions as relational systems, align and map these systems to each other, and draw inferences based on the alignments (and misalignments) for successful schema formation (see Gentner, 1983; Gick & Holyoak, 1983; Richland, Zur & Holyoak, 2007).

Orchestrating classroom lessons in which learners successfully accomplish relational structure mapping is not straightforward, particularly because opportunities for learning through structure mapping often fail in laboratory contexts (e.g., Gick and Holyoak, 1983; Ross, 1989). Specifically, reasoners regularly fail to notice the utility of aligning and mapping two or more available relational structures.

The low success rate with which participants notice and use relational structure mapping, or analogy, within laboratory studies to solve problems may in part reflect limitations in the working memory system (see Waltz, Lau, Grewal & Holyoak, 2000). Working memory is required to relationally represent systems of objects, in this case steps to solution strategies, to re-represent these systems of relations so that their structures can align and map together, to identify meaningful similarities and differences, and to derive conceptual/ schematic inferences from this structure-mapping exercise to better inform future problem solving (see Morrison, Krawczyk, Holyoak et al 2004).

The current study tests the role of visual representations of the source and target analogs within an opportunity for structure-mapping. The manipulation assesses whether 1) making source and target analogs visual (versus oral) increases the likelihood that participants will notice and successfully benefit from structure mapping opportunities, and 2) whether the visual representations must be visible simultaneously during structure-mapping in order to increase the likelihood of future success in problem solving and schema formation. The former is likely to increase the salience of the relational structure of each representation, while the latter is likely to reduce the working memory load and executive function resources necessary for participants to engage in structure-mapping and inference processes.

These are research questions with high ecological validity. A cross-cultural study of 8th grade mathematics instruction revealed that comparing verbal and visual structured representations is a common practice in U.S. mathematics classrooms as well as in higher achieving regions (Hong Kong and Japan), but that U.S. teachers are less likely to make visual representations visible during a structure-mapping episode than the teachers in higher achieving countries (Richland, Zur & Holyoak, 2007). Thus findings from this experiment will yield both theoretical insight into the resource load necessary for complex structure mapping and schema formation, and practice relevant implications for everyday mathematics teachers.

Because the study takes ecological validity and the complexity of everyday classrooms as serious constraints, a novel methodology was used to derive rigorous, experimental data that incorporates the complexities of situated cognition. Specifically, the stimuli for the experiment derive from videotapes of a public school

teacher in her naturalistic classroom, teaching a lesson co-designed with the research team. This methodology and its motivation are next explained in more detail, followed by a report of the experiment itself.

Video-editing as a Tool to Bridge Laboratory and Classroom Settings

Classrooms are vibrant, complex environments in which the high level of unexpected variability makes experimental control often impossible (Brown, 1992). The overarching commitment to controlled manipulation of experimental contexts within psychological research has led much cognitive scientific study of learning behavior to be conducted in controlled laboratory settings. While in some ways this model leads to the production of data that can be easily interpreted (x behavior derived from y manipulation), the meaningfulness of these results for educational practice have been less clear. Theoretically, this research epistemology has also meant that the search for universal cognitive processes of learning can best be accomplished through the design and examination of cognition within atypical, impoverished environments (see Schweder, 2012). The assumption that cognitive mechanisms underlying classroom learning are not moderated by environmental factors is unexplored.



Figure 1. Still images illustrating the experimental conditions created by video editing the same lesson, from left to right: Not Visible, Part Visible, and All Visible.

The current study does not interrogate that question but rather reduces the assumption by situating the stimuli creation in the naturalistic classroom context itself. A naturally occurring classroom lesson is videotaped using three cameras that capture different features of the lesson, (e.g., teacher only and teacher plus visual representations) though the same classroom discourse, affect, eye gaze, and many other potentially important features of the context are held constant across cameras. The distinct camera angles are then used to create different conditions of a videotape of the

same lesson, which are then shown to a new group of classroom students. This is clarified in the below description of stimuli creation for the current study.

Experiment: Impact of Visual Support for Instructional Analogy

Method

Participants. Participants were drawn from a suburban public school with a diverse population. Five students that scored in the bottom 5% of the participant pool were also excluded from analyses. The final analyses included 78 students (46 boys, 32 girls) with ages ranging between 11-12 years old. Within classrooms, students were randomly assigned to condition, with 25 students in the All Visible condition, 27 students in the Part Visible condition, and 26 students in the Not Visible condition.

Materials. Materials for the intervention consisted of a worksheet, a netbook, and a pre-recorded video-lesson embedded in an interactive computer program. The lesson used in the current study was developed by the authors in collaboration with a public school teacher. Three cameras were used simultaneously to videotape a classroom lesson on ratio. Ratio was chosen for this study for two reasons: (a) it is part of the common core standards for elementary mathematics instruction and (b) previous research has shown that ratio problems prompt diverse systematic student responses, useful for charting trajectories of reasoning change across the study. One camera was set to a wide shot, captured the teacher, parts of the classroom, and all visual representations of the three solution analogs throughout the lesson (All Visible -AV). A second camera was more tightly focused, capturing the teacher, some of the class, and only the visual representation of a solution as it was being produced (Part Visible - PV). The third camera focused only on the teacher and students, and did not capture any of the visual representations of the solutions written onto the white-board (Not Visible - NV; see Figure 1 for an illustration of each condition).

The video-lesson was made interactive by embedding clips of the video in a computer program. These stimuli were then used experimentally with students in other classrooms. This methodological approach of stimuli creation, provided a rigorous level of experimental control of a highly dynamic context – an everyday classroom. Further, it allowed for randomization within each classroom, which to the authors knowledge has not been previously done using a video teacher’s guidance.

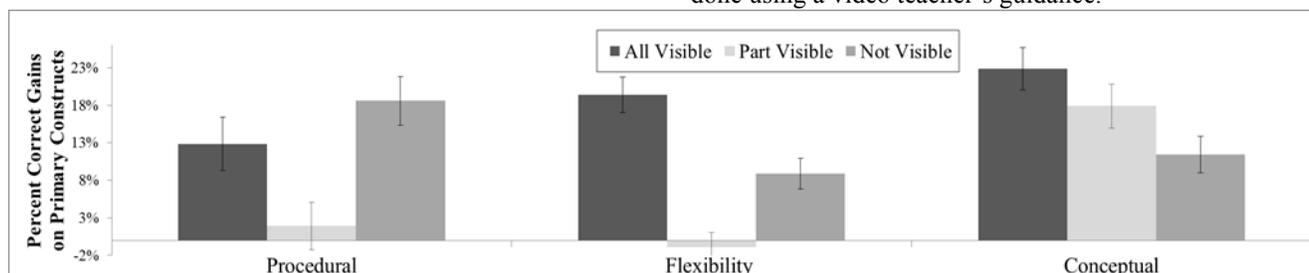


Figure 2. Gain scores for immediate and 1-week delayed posttest calculated by subtracting mean pretest score with respective posttest score.

Table 1. Mean Gain Scores by Knowledge Type for Each Construct Calculated by Subtracting Pretest from Respective Posttest.

Knowledge Type	Procedural		Flexibility		Conceptual		Negative Transfer		Misconception		Efficient Strategy	
	Immediate	Delayed	Immediate	Delayed	Immediate	Delayed	Immediate	Delayed	Immediate	Delayed	Immediate	Delayed
All Visible	13%	20%	19%	27%	23%	25%	-28%	-24%	-7%	-10%	35%	45%
Part Visible	2%	-1%	-1%	7%	18%	13%	-10%	-7%	9%	12%	17%	13%
Not Visible	19%	27%	9%	21%	11%	18%	-19%	-14%	-2%	-3%	20%	33%

Assessment. The assessment was designed to assess schema formation and generalization. Mathematically, the assessment included three constructs, procedural knowledge, flexibility, conceptual knowledge, and negative transfer. The first three constructs were conceptually derived from Rittle-Johnson and Star (2007; 2009), and adapted to the core concepts and procedures underlying ratio problems (Figure 3). Scores for each construct were averaged to yield an overall mean for that particular construct.

Ken and Yoko shot several free-throws in their basketball games. The result of their shooting is shown in the table. Who is *better* at shooting free throws?

	Shots Made	Total Shots Tried
Ken	12	20
Yoko	16	25

Adelina and Marcos have both set up lemonade stands. Adelina's lemonade recipe uses 2 cups of lemon juice and 1 cup of water. Marcos' lemonade recipe uses 3 cups of lemon juice and 2 cups of water.

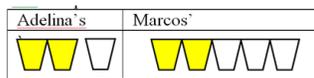


Figure 3. Procedural/Procedural flexibility problem (left) used in the video-lesson and assessments, and conceptual problem used in assessments. For procedural flexibility students were told to solve a problem similar to the one on the left using two different strategies.

Procedural Knowledge. Procedural problems on the pretest evaluated whether students had the basic skills necessary to solve ratio problems and designed to test students' knowledge of producing solutions of familiar and transfer problems. Cronbach's alpha was .89 at posttest, .92 at delayed posttest, and .86 at pretest.

Procedural Flexibility. The procedural flexibility construct measured: (a) students' adaptive production of solution methods (n=3), (b) their ability to identify the most efficient strategy (n=1), and (c) students' ability to identify a novel solution method which was related to a taught strategy (n=1). Cronbach's alpha on the flexibility construct was .67 at posttest, .67 at delayed posttest, and .57 at pretest.

Conceptual Knowledge. The conceptual construct was designed to probe into students' explicit and implicit knowledge of ratio. Cronbach's alpha was .66 at posttest, .64 at delayed posttest, and .42 at pretest.

Negative Transfer. The purpose of the negative transfer construct was to measure whether students will overextend their knowledge of ratio to similar looking problems for which a strategy shown to be invalid during instruction – subtraction, is correct. While this construct was expected to assess overextensions of the taught strategy, due to its high similarity with the taught problems, it can also help diagnose whether conditions that do not eliminate the misconception appear to be sensitive to variations in the

problem type. Cronbach's alpha was .68 at posttest, .81 at delayed posttest, and .58 at pretest.

Efficient Strategy. The aim for this measure was to assess learners' ability to utilize the most efficient solution as instructed during the video lesson. This has also been called adaptive choice of strategy (Siegler, 1996). Efficient strategy was assessed by scoring all problems taking the form of the problem taught in the video lesson to evaluate whether students used the most efficient strategy taught - the division method.

Common Misconception. Misconceptions are mistakes that students make, which obstruct learning (Smith, diSessa, Roschelle, 1994). Based on a published lesson (Shimizu, 2003), pilot data, and pretest data, a solution involving subtraction was expected to be the most common misconception participants would bring to the study. This score assessed students' ability to overcome their misconceptions about how to solve rate and ratio problems as well as the conditions under which students confirm invalid biases. The common misconception measure examined students' use of subtraction by scoring problems that looked like the instructed problem in the video lesson.

Design & Procedure. Students who were not in the original classroom lesson interacted with videotaped lesson clips via computer. The study followed a standard experimental procedure (pretest, intervention, immediate posttest, and 1-week delayed posttest). Students within a classroom were randomly assigned to either watch an instructional video version video-edited so that no solutions were visible on the board – Not Visible (n = 26), a version where the most recent solution was visible – Part Visible (n = 27), or a version of the video that showed all solutions on the board throughout the lesson - All Visible (n = 25). All students were given a packet on which they recorded their answers to prompts from the videotaped lesson. Students underwent the intervention before being introduced to rate and ratio.

Results

First, between-subjects regression analyses revealed no differences between conditions on any of the outcome constructs. Boys and girls also did not differ in performance. Separate univariate one-way between-subjects ANCOVAs were run for each construct with posttest or delayed posttest as a dependent variable and pretest as a covariate. Average gain scores at posttest for the main constructs are shown in Figure 2, and the full set of gain scores immediately and after a delay are provided in Table 1. Table 2 provides all statistics, revealing that the All Visible condition outperformed the Part Visible condition in procedural knowledge, procedural flexibility, conceptual knowledge and efficient strategy both on immediate and

Table 2. Between Group Comparisons for Each Construct on Immediate and Delayed Posttest. *p-values* * > 0.10, ** > 0.05, *** > 0.01

Knowledge Type	Procedural		Flexibility		Conceptual		Negative Transfer		Misconception		Efficient Strategy	
	Immediate	Delayed	Immediate	Delayed	Immediate	Delayed	Immediate	Delayed	Immediate	Delayed	Immediate	Delayed
All vs. Not Visible	0.646	0.951	0.041**	0.250	0.038**	0.338	0.117	0.308	0.095*	0.171	0.307	0.297
All vs. Part Visible	0.027**	0.019**	0.000***	0.021**	0.073*	0.030**	0.014**	0.104	0.018**	0.000***	0.001***	0.000***
Not vs. Part Visible	0.079*	0.026**	0.176	0.150	0.750	0.325	0.377	0.530	0.498	0.027**	0.117	0.014**

delayed posttests. The inverse was true for the negative transfer on immediate posttest and common misconception construct on both posttests, which is indicating that most students are being misled by the appearance of the problems (which are similar to the ratio problems) and assume they are solving for a ratio problem, not a simple subtraction problem. Thus, PV students may have used the misconception throughout all problems, regardless of whether it was correct or not. Students in the All Visible condition also outperformed students in the Not Visible condition in the flexibility and conceptual knowledge constructs on the immediate posttest. For AV and NV students the differences do not seem to hold at a delayed posttest. The NV students were better than PV students when measured for use of efficient strategy and common misconception, after a 1-week delay.

Discussion

The results of this study clarify the role of visual representations in supporting structure-mapping and generalization from instructional analogies. The manipulation revealed that making source and target analogs visual (versus oral) increased the likelihood that participants notice and benefit from structure mapping opportunities. As noted above, the use of a visual representation of a structured relational analog was hypothesized to be likely to increase the salience of the relational structure of each representation, while maintaining their visibility was predicted to reduce the working memory load and executive function resources necessary for participants to engage in structure-mapping and inference processes.

The data revealed that this variation in visible representations had a great impact on their learning. Seeing all problem solutions on the board simultaneously during structure-mapping led to the most robust and generalizable, flexible knowledge acquisition in the context of this intervention. Having all visual representations available throughout the lesson may provide students with the necessary working memory supports to attend to key elements in the source and target representations, enabling the child to represent the solutions as systems of relations, map these representations together, and correctly identify elements that are in alignment (or misalignment). Thus, children in the AV condition may have successfully accomplished and benefitted from structure-mapping, while children in the PV and NV conditions may have benefitted less from the instructional analogy itself, though both groups did show knowledge acquisition. This may explain the AV students' greater gains in flexible use of strategies

and conceptual knowledge, compared to PV students (on both posttests) and NV students (on immediate posttest).

These data coalesce with results from Rittle-Johnson and Star (2007; Star & Rittle-Johnson, 2009), who administered measures of procedural flexibility and conceptual knowledge with pre-algebra concepts and found that having pairs of students compare representations simultaneously was more effective than sequentially for students with adequate entry level knowledge of estimation strategies. The current study provides specificity to instructional techniques and supports previous findings with more ecologically valid stimuli, but also provides more detailed data on the role of visual representations of source and target analogs.

The data for the implications of constructing visual representations of analogs but not leaving them visible throughout the lesson are quite different. The least flexible learning derived from the PV condition. This may be because the use of the visual representation did draw learners' attention to the structure of the discussed solution representations, but these learners did not have the resources to move beyond these representations to perform structure-mapping and schema generalization.

In contrast, the NV participants may not have had the executive function and working memory resources available for complex structure mapping between representations, they may have also encoded less of the lesson and the first solution (a misconception), may have been less instantiated for them. The delayed data support this interpretation. While the difference in learning gains between AV and PV students remained after one-week delay, this was not so when comparing AV and NV students, highlighting that lack of visual information was less detrimental to overall learning than providing one visual representation at a time.

In fact, students in the Not Visible condition outperformed students in the Part Visible condition in procedural knowledge significantly at a delayed test, and this difference approached significance at immediate posttest. Perhaps, keeping only the latest representation visible on the board may be detrimental for teachers looking to challenge students' misconceptions. Students who see a instantiate a solution modeled on the board as valid, particularly if it is easier, (e.g. subtraction is easier than division), despite teachers' efforts to show it is incorrect.

Previous research suggests that students seek to validate their misconceptions (Chinn and Brewer, 1993) and having the misconception visible, but not throughout the entire lesson in which it was compared to two more accurate solutions, may have helped students in doing that, even more than if it was never visible. Understanding the cognitive processes at play that reconcile these results

warrants further investigation. Outcomes in negative transfer and common misconception provides further support that seeing one problem at a time is detrimental for students attempting to learn by drawing connections between solution strategies.

A reverse trend is apparent for students overextending instructed strategies to problems appearing similar to taught problems, but where the common misconception is the correct solution strategy. The results for the negative transfer measure show that PV students outperform AV students on immediate posttest, which may have been a result of the PV students using the misconception as a correct strategy for all problems that appeared like the problem used in the lesson. This is supported by the fact that PV students also used the common misconception more than AV students on both posttests. Recall that in the video lesson, the common misconception was modeled and was discussed only to be exemplified as an invalid strategy. Challenging misconceptions by modeling and discussing them is common practice in higher achieving countries (e.g. Japan and Hong Kong) and recommended by researchers as a way for students to overcome them (Berry and Graham, 2006; Kuhn, 1989). However, students who did not see the misconception compared with valid solution strategies, despite hearing the same verbalization, may have failed to overcome this challenge and instead may have led them to memorize the misconception as a valid strategy.

Zook (1991) conceptualizes two factors that may be responsible for developing misconceptions from analogies. The first is learner-generated and the second is teacher-generated, either leading to misconceptions, which Zook (1991) defines as: (a) difficulties of the learner attending to key elements increases the potential for misconceptions, and (b) difficulties in leading learners' attention to key elements increases the potential for misconceptions. An interplay of these factors may have negatively affected students in the Part Visible condition in their procedural knowledge, because the valid solutions were not visible throughout the lesson. From the teacher's perspective, there were not sufficient visual cues to support the verbal explanations provided in the instruction, so, from a students' perspective, students had difficulty attending to key mathematical ideas necessary to overcome their misconceptions.

Misconceptions are common throughout the curriculum, and researchers focused on the potential of analogies to overcome these through conceptual change have revealed the real challenges of teaching children to reconsider their misconceptions. For example Chinn and Brewer (1993) provide evidence that many students finish high-school and University without giving up pre-Newtonian perspectives of motion (e.g., Clement, 1982).

Overall, teaching through instructionally supported structure-mapping has the potential to enhance students' conceptual knowledge, procedural flexibility, and procedural knowledge in mathematics. Visual representations can augment these benefits, though it is important to note that the overall advantages in procedural

flexibility in this study were driven by students who saw all the solutions on the board at all times, where students who saw only one solution at a time did most poorly. Strikingly, for procedural knowledge, students who only saw one solution at a time performed worse than students who did not see any solutions throughout the lesson. Thus, students in the Part Visible and Not Visible condition may not have learned by analogy due to insufficient supports.

Implications for Theory and Practice

The findings from this study have the potential to positively shape U.S. teaching practices as well as contribute to several areas of cognitive scientific literatures. Utilizing teaching by comparison is critical for learning deep mathematical conceptual knowledge (Rittle-Johnson and Star, 2007, 2009; Star and Rittle-Johnson, 2009; National Mathematics Panel, 2008a, 2009b). Teachers in the U.S. rarely scaffold instructional comparisons adequately (Richland, Zur, and Holyoak, 2007; Heibert et al., 2005), this has been partly due to a lack in specificity in recommendations on how to improve these practices (Hiebert et al., 2005). Recent work that has used cognitive science research in the classroom has provided positive evidence in this direction (Rittle-Johnson and Star, 2007, 2009; Star and Rittle-Johnson, 2009), but even these studies do not examine instructional strategies as they unfold in a real classroom lessons.

The current study uses a novel theoretical perspective and methodological approach that bridges analogy research in laboratory settings with studies of instructional practice in classroom environments. From a theoretical standpoint, these findings support previous laboratory-based results indicating that visual representations can support schema formation and learning from analogy (Gick and Holyoak, 1983), and extend them to an applied setting. The current study provides data on a relevant instructional scaffold that facilitates learners' ability to draw connections between mathematical solution strategies. Comparing representations is common to everyday mathematics instructions, and making all source and target representations visible for the length of the analogy requires only a small time investment and modification of current practice. Thus this intervention is highly feasible to integrate into current teaching practices. Using more ecologically valid stimuli to test teaching practices with the use of a videotaped teacher guided lesson, instead of static written learning materials, thus allows for greater generalizability and specificity for instructional recommendations. Further research that uses these experimental methods is underway, and the authors encourage interdisciplinary researchers to consider the use of video stimuli when designing educational studies.

One must note that we cannot interpret these results to indicate that making analogs visible simultaneously will necessarily lead to successful structure-mapping and mathematical schema formation. Key to improving educational practice is certainly ensuring the instruction uses optimal structured analogs, and ensuring that any misconceptions are identified and compared well with an

alternative and more accurate representation.. Much is still unknown about the ideal combination of support for instructional analogy. At present, further studies are being conducted to examine the impact of the following practices: (a) the teacher's gestures when presenting and linking key ideas, (b) the visual organization of solutions on the board, and (c) the sequencing of chosen solutions (i.e. beginning with a common misconception versus a correct strategy). The first of these two practices (a) and (b) were observed by Richland, Zur, and Holyoak (2007) to correlate with the practices used in our experiment, but they remain to be tested experimentally.

Lastly, from a technology perspective, these findings could have implications for current trends to replace classic chalk or white-boards with smartboards, the highly popular electronic innovations that enable teachers to use their board very actively as a dynamic connection to their computer. While there is the potential for rich activity, there is little room to write, since the smart boards are about a third of typical white boards. These data suggest that smartboards may be highly effective at instantiating single visual representations at a time, much as in our part visible condition, which led to the lowest learning gains, greatest rate of misconceptions, and least flexible knowledge. In summary, instructional attention should be paid to carefully considering the role of visual representations in balancing the benefits for improved encoding of relational structure with ensuring that students align, map, and compare these structured representations to ensure broader generalization, misconception revision, and appropriate schema formation.

References

- Berry, J. and Graham, T. (1991). Using concept questions in teaching mechanics. *International Journal of Mathematical Education in Science and Technology*, 22:5, 749-157.
- Brown, A. L. (1992) Design Experiments: Theoretical and Methodological Challenges in Creating Complex Interventions in Classroom Settings. *The Journal of the Learning Sciences*, 2(2), 141-178.
- Good, T and Brophy, J. (2003). Looking in classrooms (9th ed.) Boston: Allyn and Bacon.
- Chinn, C., & Brewer, W. (1993). The role of anomalous data in knowledge acquisition; A theoretical framework and implications for science instruction. *Review of Educational Research*, 63(1), 1-50
- Clement, J. (1982). Students' preconceptions in introductory mechanics. *American Journal of Physics*, 50, 66-71.
- Gentner, D. (1983). Structure-mapping: A theoretical framework for analogy. *Cognitive Science*, 7, 155-170.
- Gentner, D. (2010). Bootstrapping the mind: Analogical processes and symbol systems. *Cognitive Science*, 34, 752-775.
- Gick, M.L. and Holyoak, K. J. (1983). Schema induction and analogical transfer. *Cognitive Psychology*, 15, 1-38.
- Good, T and Brophy, J. (2003). Looking in classrooms (9th ed.) Boston: Allyn and Bacon.
- Hiebert, J. Stigler, J. Jacobs, J., Givvin, K. B., et al. (2005); *Mathematics Teaching in the United States Today (and Tomorrow): Results From the TIMSS 1999 Video Study Educational Evaluation and Policy Analysis*.
- Holyoak K. J. (2012). Analogy and relational reasoning. In *Oxford handbook of thinking and reasoning*, pages 234-259. Oxford University Press.
- Morrison, R. G., Krawczyk, D. C., Holyoak, K. J., Hummel, J. E., Chow, T. W., Miller, B. L., et al. (2004). A neurocomputational model of analogical reasoning and its breakdown in Frontotemporal Lobar Degeneration. *Journal of Cognitive Neuroscience*, 16, 1-11
- Morrison, R. G., Doumas, L. A. A., and Richland, L. E. (2011) A computational account of children's analogical reasoning: Balancing inhibitory control in working memory and relational representation. *Developmental Science*, 14, 516-529.
- National Mathematics Panel, (2008). *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*, U.S. Department of Education: Washington, DC.
- National Research Council. (2001). *Adding it up: Helping children learn mathematics*.
- Richland, L.E., Zur, O., and Holyoak, K.J. (2007). Cognitive supports for Analogies in the Mathematics Classroom. *Science*, 316, 1128-1129.
- Rittle-Johnson, B, and Star, J.R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology*, 99, 561-574.
- Rittle-Johnson, B., and Star, J. R. (2009). Compared to what? The effects of different comparisons on conceptual knowledge and procedural flexibility for equation solving. *Journal of Educational Psychology*, 101, 529-544.
- Ross, B.H. (1989). Distinguishing types of superficial similarities: Different effects on the access and use of earlier problems. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 15, 456-468.
- Star, J. R., and Rittle-Johnson. B. (2009). It pays to compare: An experimental study on computational estimation. *Journal of Experimental Child Psychology*, 102, no. 4: 408-426.
- Stigler, J., and Hiebert, J. (1999). *The Teaching Gap*. NY: The Free Press, Simon and Schuster, Inc.
- Smith J.P., III, diSessa A. A., and Roschelle J. (1994). Misconceptions Reconceived: A Constructivist Analysis of Knowledge in Transition. *The Journal of Learning Sciences*. 3(2), 115-163.
- Waltz, J.A., Lau, A., Grewal, S. K., Holyoak, K.J. (2000). The role of working memory in analogical mapping. *Memory and Cognition*, 28(7):1205-12.
- Zook, K. B. (1991). Effects of analogical processes on learning and misrepresentation. *Educational Psychology Review*, 3, 41-72.